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Investigating problem-solving behaviours of university students through an eye-tracking system using GeoGebra in geometry: A case study

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Abstract

Problem solving is one of the high-level thinking skills and essential in teaching mathematical concepts and procedures. The eye-tracking method allows educators to see and interpret different problem-solving behaviours of students. The purpose of this study was to investigate the problem-solving behaviours of university students based on eye tracking statistics when faced with a geometry problem, and to explain these behaviours in terms of the students' success and learning styles. The research design was a case study approach that combined quantitative and qualitative data. Eight first-year students from the department of elementary mathematics education in a private University, Turkey, participated in this study. They were asked to solve a specific geometry question using the GeoGebra software. While solving this question, their eye movements were recorded and analysed using an eye-tracking program. Results were analysed to investigate their problem-solving skills and behaviours based on Polya's mathematical problem-solving stages. Results of the study suggest that the effort devoted to the understanding and planning stages of problem-solving are important factors that contribute to success in this case. Results also revealed that participated students with a converger learning style were more successful in each step of the problem-solving process. Additionally, it could be concluded that the input area had only been used by converger students, indicating that they devoted time and effort to creating and trying formulas.

Keywords Eye tracking · Problem solving · Learning styles · GeoGebra · Geometry

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1 Introduction

Problem solving is a vital skill that plays an important role in almost all levels of education for several reasons. During the problem-solving process, a) students build new mathematical knowledge, b) reflect on their thinking, c) apply and adapt strategies and d) solve other problems in other contexts (National Council of Teachers of Mathematics [NCTM], 2000).

The problem-solving process is extensive in itself. Dede and Yaman (2006) identified the stages of problem solving: recognising, encountering and revising. In addition, Polya's (2004) mathematical problem-solving stages are among the best-known methods in the field. These stages include i) understanding the problem, ii) planning, iii) applying and iv) revising. These stages are commonly referred to as Polya's four-step problem-solving method and are widely used in mathematics education and problem-solving in other fields.

Polya (2004) pointed out that the way of looking at a problem changes repeatedly throughout the problem-solving process. He emphasized that our initial understanding of the problem is often incomplete, and our outlook changes as we make progress and approach a solution. Polya highlights the importance of each stage in successfully solving a problem. To explain these four stages operationally, Polya (2004, p.5) states, "First, we must understand the problem and clearly see what is required. Second, we must determine how the various components are connected, how the unknown is linked to the data, in order to develop an idea of the solution and formulate a plan. Third, we execute our plan. Fourth, we review and discuss the completed solution."

Polya's (2004) four-stage model provides a framework for understanding the problem-solving process. However, to gain a more comprehensive understanding of this process, some researchers also have turned to eye tracking. By examining the eye movements, fixation patterns, and gaze transitions of students as they work through problems, researchers can gain valuable insights into the cognitive processes that underlie effective problem-solving. This information can be used to develop more effective teaching strategies and to identify students who may benefit from additional support in developing their problem-solving skills.

1.1 Eye-tracking technology in understanding the problem-solving process

Eye-tracking is one of the emerging technologies widely used to analyse student behaviour and personalizing learning for students (Seo et al., 2021) in digital educational environments. It is a method that tracks and records users' eye movements on a screen, measuring the duration of focus on different areas. This makes it possible to track student progress as a given task is completed. Such a system could provide information regarding strategies followed and time devoted to certain tasks.

In education, eye-tracking methods have been used to investigate students' problem-solving strategies in several disciplines, including mathematics. In such a way, information on student behaviours while problem solving in mathematics could be

observed in detail. In the literature, there are eye-tracking studies that were conducted to determine the problem-solving strategies of students when solving mathematical problems (e.g. Lin & Lin, 2014; Schot et al., 2015; Zheng & Cook, 2012) and learning (e.g. Bolzer et al., 2015; El Haddioui & Khaldi, 2012; Mayer, 2010).

The results of the studies by Hegarty et al., (1992, 1995) indicated that students' eyes focus on particular areas while they are solving problems. Those with more correct answers focused more on difficult questions as well as in planning and application steps of problem solving. In their study of the impact of learning in mathematics, Andra et al. (2015) examined the differences in eye movements of students on varied mathematical representations of the same mathematical object. The results indicated a meaningful difference in eye movements between formulae and graphs.

In another study, Sajka and Rosiek (2015) found that time to analyse the wording of a problem and the number and placement of fixations (which are the most common eye movement thorough concentrating on a single spot, can provide insight into cognitive and processes and attention) were two factors that significantly differentiated mathematically gifted students from their peers. This study also proved that the total amount of time taken to solve a problem and speed were not important differentiating factors.

Bolden et al. (2015) also used an eye-tracking method to investigate the problem-solving skills of students. One of the important results of that study was the role of eye-tracking technology in investigating young children's focus of attention while undertaking a mathematics assessment.

In the study of Hu et al. (2017), it was found that high-performing students had long durations of fixation in the problem-solving stage of the interactive problem. In the problem exploration stage of the interactive problem, high-performing students had short fixation duration. These findings demonstrate how students had varying behaviours for different types of problem-solving scenarios at different problem-solving stages.

In order to be successful with problem-solving strategies, it is important to spend enough time on a task to understand the problem. Problem-solving strategies in which the learner spends more time looking at the task itself is a more successful strategy (Dobias et al., 2017).

Tóthová et al. (2021) suggested that the time spent on solving the problem or completing the task has no meaningful relationship with successfully solving a problem or completing a task. However, as the difficulty level of a task increases, the time devoted to the problem also increases.

Lin and Lin (2018) used eye tracking and a handwriting device to study geometry problem-solving. Unsuccessful students in the study had longer dwell times (DT), higher fixation counts (FC), and increased run counts (RC). When dealing with adjacent triangles, these students struggled with frequent fixation on known and unknown variables.

Schindler and Lilienthal (2019) studied eye movement patterns during geometry problem-solving with one participant. Their findings confirmed the eye-mind hypothesis, indicating a strong connection between what is fixated and what is mentally processed. In essence, the study unveiled insights about what drew the

participant's attention, where they concentrated, and why they revisited certain areas, shedding light on various mental processes.

Eye-tracking technology can be used to understand how students approach and acquire geometric concepts (Uygun et al., 2022). Eye tracking measurements can serve as an important predictor for academic achievement in the context of mathematics in geometry. They suggested that eye measurements could be used as an indicator to gauge students' problem-solving difficulties, with longer eye fixation durations indicating greater difficulty (Simsek et al., 2020).

A review of eye tracking studies in mathematics by Strohmaier et al. (2020) revealed that eye tracking is especially useful in mathematics research for studying processes, revealing mental representations, and assessing subconscious elements of mathematical thinking, rather than focusing solely on outcomes.

There has been a growing interest in using cognitive approaches to investigate learning in mathematics through eye trackers. In a recent study by Schindler and Lilienthal (2022), researchers investigated the collaborative creative process in mathematics by observing two students working together on a multiple solution task. They found that the collaborative process was similar to solo creative work in some ways, but it was also more complex and dynamic. The findings of this study can be used to develop new educational programs and teaching methods that promote collaborative learning in mathematics.

Eye tracking studies in geometry have found that cognitive processes can be measured by specific eye tracking data. Learning can be explained by this data, but it cannot fully explain it. Other methods, such as think aloud (Elbabour et al., 2017) and portable devices (Pavlov et al, 2022) can be used to support the eye tracking data to help elucidate students' cognitive abilities. For example, Alqassab et al. (2018) used total dwell time (in seconds) as a measure of cognitive processing during peer feedback provision. Transitions from the text to the figural components of the peer solution were also used as a measure of cognitive processing. These transitions indicate the degree to which the learner integrates information from the text and the figure. Muldner and Burleson (2015) compared the eye tracking measures of students who they grouped according to their high and low creativity levels based on their responses to a geometry question. They found that low creativity students had: marginally fewer total fixations.

In the study of da Silva Soares et al. (2023) concluded as; eye-tracking technology is becoming increasingly popular in educational research. It allows researchers to study problem-solving strategies, visual attention, and engagement in realistic classroom situations. Eye movement data can help teachers identify performance characteristics that lead to instructions regarding how a struggling student approaches maths problems. Through future use of eye tracking, they stated that it can be used as a tool for metacognition development in students and teachers and they proposed that such information could offer insights and resources for self-directed learning methods.

In this section, several studies examining the mathematics learning process using eye-tracking technology have been summarized. As a result, eye-tracking technology demonstrates that students exhibit different behaviours for different

problem-solving scenarios in mathematics learning and spend enough time on successful problem-solving strategies is crucial.

These findings demonstrate the potential benefits of using eye-tracking technology in studying the problem-solving process and inform the development of effective problem-solving strategies. However, in studying problem-solving skills via eye tracking, a different approach that focuses on the characteristics of learners such as their learning styles beyond their success is also important to understand this process better. So, as part of this study, we aimed to examine university students' problem-solving behaviours not only in terms of their success but also in relation to their learning styles.

1.2 Eye tracking technology in investigating individuals differences

Learning is a process that includes many stages, and there are multiple factors that affect one's learning. That is why each individual will show different characteristics in such a process (Özden, 2020). One of the factors that results from the learner's individual characteristics is the way he/she learns, which is known as their learning style. Keefe (1979) reported that learning style represents: "the characteristic cognitive, affective and physiological behaviours that serve as relatively stable indicators of how learners perceive, interact with and respond to the learning environment".

Since it was first published in 1976, Kolb's Learning Style Inventory (LSI) has been used extensively in academic and professional settings to identify the learning style preferences of different groupings. The Kolb (1984) LSI is based on Kolb's experiential learning theory which defines learning as "the process whereby knowledge is created through the transformation of experience. Knowledge results from the combination of grasping and transforming experience". Theory suggests that learning occurs through a cycle of four stages: concrete experience (doing/having an experience), reflective observation (reviewing/reflecting on the experience), abstract conceptualization (concluding/learning from the experience), and active experimentation (planning/trying out what you have learned). According to Kolb (1984) experiential learning theory, effective learning is seen when a person progresses through a cycle of four stages (Fig. 1): (1) having a concrete experience followed by (2) observation of and reflection on that experience which leads to (3) the formation of abstract concepts (analysis) and generalizations (conclusions) which are then (4) used to test hypothesis in future situations, resulting in new experiences (McLeod, 2017).

Individuals have a preferred learning style that is a combination of these four stages (Kolb, 1985). The LSI consists of a questionnaire that assesses an individual's learning style preferences based on four dimensions: concrete experience (CE), reflective observation (RO), abstract conceptualization (AC), and active experimentation (AE). The questionnaire includes a series of statements and asks individuals to indicate how strongly they agree or disagree with each statement. Based on their responses, individuals are categorised into one of four learning style types: diverging, assimilating, converging, or accommodating (Fig. 2). Each learning style type is associated with a unique combination of the four dimensions (Kolb, 1985).

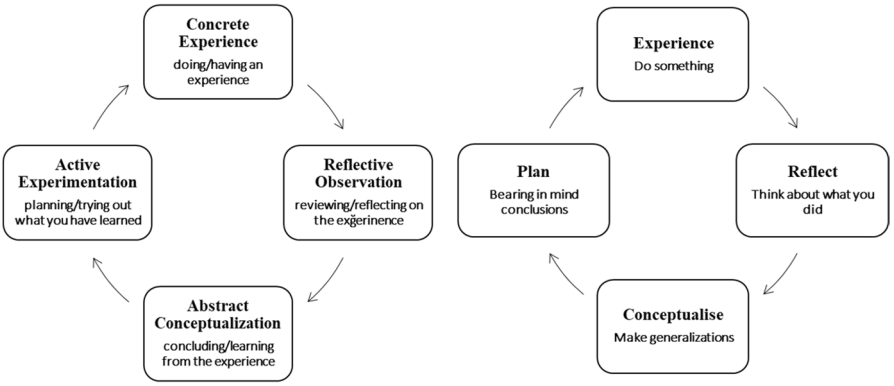

Fig. 1 Kolb's experiential learning cycle. Adapted from McLeod (2017)

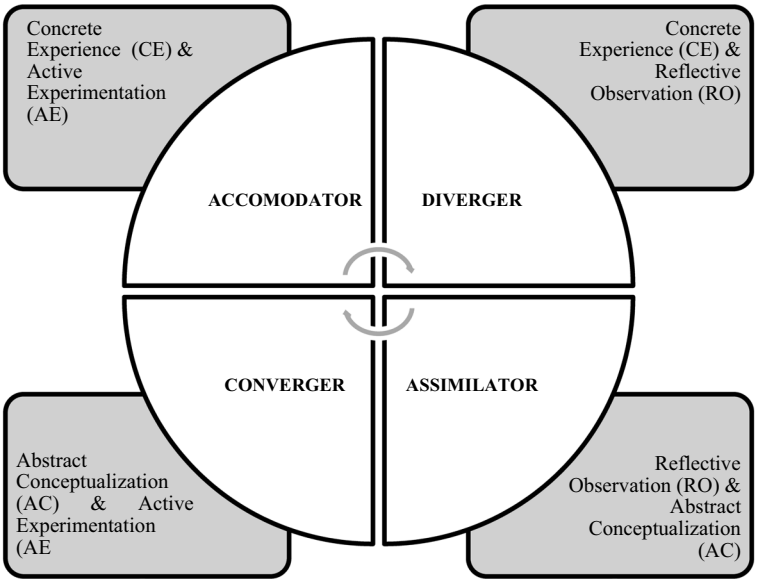

Fig. 2 Kolb's learning styles and cycles. Adapted from McLeod (2017)

Diverging learners (CE/RO) tend to be imaginative and creative, and they enjoy observing and collecting information. They are good at brainstorming and generating new ideas, but they may struggle with decision-making. Assimilating learners (AC/RO) prefer to work with abstract concepts and ideas. They enjoy reading and listening to lectures and are skilled at organising information and making connections between ideas. Converging learners (AC/AE) are practical and logical, and they enjoy solving problems and making decisions. They excel at applying theoretical concepts to real-world situations, but they may struggle with brainstorming and generating new ideas. Accommodating learners (CE/AE) are

hands-on and prefer to learn through trial and error. They enjoy taking risks and trying new things, but they may struggle with abstract concepts and theoretical ideas (McLeod, 2017).

Although few in number, there are studies that have researched the learning styles of students through eye-tracking systems (e.g. Catrysse et al., 2018; Papaeconomou et al., 2008; Tsianos et al., 2009). Najar et al. (2015) also used eye-tracking technology to identify differences between learners.

Rohmanawati et al. (2021) concluded that Kolb's (1985) learning styles greatly affect students' mathematical communication skills. Their results indicated that students with the assimilator type of learning style 'more mastered in terms of describing the problem situation in the form of geometry and using mathematical solutions related to the analysis of other forms while the subject with accommodator type of learning styles more mastered in investigating what is known and describe the problem situation in the form of geometry'.

Related to the different behaviours of students with different learning styles based on Kolb's (1985) classification, Rahmah et al. (2022) concluded that the subjects with assimilator and converger learning styles could solve problems correctly from the action (step by step procedure on how to perform operation) and process (establishment of the mental construction internally) while they could not able to establish object stage (obtaining the result of mental construction).

Tsianos et al. (2009) conducted an eye-tracking experiment to validate the concept of learning styles as a personalisation parameter in adaptive e-Learning systems. Their findings revealed statistically significant differences among the separate types of learners: imagers concentrate on visual content, verbalisers focus on text and intermediates use both to a similar degree.

Learning styles are important teaching and learning phenomena that play an important role in the development of mathematical concepts in students (Özerem & Akkoyunlu, 2015).

This study builds upon previous research that has identified the importance of eye-tracking technology in investigating university students' focus of attention during problem-solving tasks. Our search of the literature revealed a need for studies that focus on mathematical problem-solving skills of students on a specific task in geometry in terms of their performance and learning styles. The purpose of this study was to investigate the mathematical problem-solving behaviours of students based on i) eye tracking statistics and ii) observation of students through screen and outer camera recordings and iii) explain such behaviours in terms of the success and learning styles of students. In this study, categorisation of the main attributes was based on Polya's (2004) three problem-solving stages (understanding, planning, solving) and Kolb's learning styles (convergers vs assimilators) providing a more nuanced understanding of how students with different performance and learning styles approach and solve mathematical problems. The revision stage is not included in this study as explained in the method. Overall, this study contributes to the growing body of research on the role of eye-tracking in investigating problem-solving behaviours based on students' performance and learning styles in mathematics education.

2 Method

The purpose of this study was to investigate the problem-solving skills/behaviours of university students in terms of their success/performance and learning style. To do this, we used dynamic geometry and an eye-tracking software. We used a qualitative case study design with a mixed methods approach in an embedded design. In such an approach, users collect and analyse both types of data at the same time within a larger quantitative or qualitative design in which one type of data is secondary to the other (Creswell & Clark, 2007; Johnson & Onwuegbuzie, 2004). The data obtained from the eye-tracking records of students were analysed using mainly quantitative and supplementary qualitative methods.

2.1 Participants

A purposive sampling method was used. Purposive sampling provides the opportunity to research deeply by selecting situations rich in the data needed to address the research aim. This sampling method is appropriate when the aim is to study individuals or a group with certain characteristics (Büyüköztürk et al., 2015). This study involved twelve university students from the department of mathematics education at a private university in Turkey. Students were chosen who had equal experience using GeoGebra and similar final course grades (85–100) from past semesters' geometry course. (In that past course students were introduced to GeoGebra for the first time. Throughout the term, students' final geometry grades were determined by weighting their midterm grades with the final exam). Students also declared their level of GeoGebra usage in the questionnaire in the demographic information part. Four students were excluded because of eye calibration problems during the study. Finally, eight voluntary prospective mathematics teachers (all women; five convergers, three assimilators) from the department of Elementary Mathematics Education at a university in Ankara, Turkey, participated. Codes (K1, K2, etc.) were given to each student to preserve their anonymity. Student characteristics are reflected in Tables 2, 3, 4, 5, 6, and 7 in the results sections for better understanding of each situation in light of the eye-tracking statistics.

2.2 Procedure

GeoGebra is a dynamic software that combines geometry with analysing processes (Hohenwarter & Jones, 2007). Such a dynamic geometric system helps students solve geometry problems and instructors teach important concepts in geometry.

In this study, problem-solving behaviours of university students based on Polya's (2004) theory were investigated while students worked at the GeoGebra interface. It was decided to include only three of Polya's problem solving stages: understanding, planning, and solving. The revision stage was not included in the eye-tracking analysis. Main reason for this was to eliminate the possible increase in the number of fixation counts by adding a fourth stage (revision stage). This would cause confusion

among the other three previous stages by changing that fixation counts during go backs in the revision stage. Another reason was to narrow the focus of study and to concentrate deeply on the three stages to have the clearest picture and explanation of the cases as possible. Eight students tried to solve a given problem independently at an eye-tracking laboratory. Eye-tracking application was conducted after class hours in the laboratory. There was no time limit per student, but a maximum of two hours was suggested as sufficient to complete the solution. First, a pilot study was conducted with two students who were not participants of the study, where the principles of Tobii Studio were experienced and the framework for the determination of video recordings was realised. Students were requested to develop solutions to the problem presented using the GeoGebra interface. The task was 'Proof that the angle of a triangle in a semicircle is 90°'.

2.3 Data collection tools

The data collection tools used to obtain information from the participants are given in each paragraph.

2.3.1 Kolb's learning style ınventory

As also stated in Kolb and Kolb, (2005b), "LSI differs based on experiential learning theory (ELT) and differs from other tests of learning style and personality used in education by being based on a "comprehensive theory of learning and development". This distinction is explained based on five different levels as; personality, educational specialization, professional career, current job role and task skills. They stated patterns of behaviour that are associated with four basic learning styles are shaped by transactions between people and their environment at these five different levels. So, it is important to note that in our study, we used Kolb's inventory in such a way to determine students' learning styles in their current situations at five levels. We also considered other student characteristics, such as academic standing and experience with GeoGebra, in our results.

Kolb's (1984) learning style inventory was adapted into Turkish by Aşkar and Akkoyunlu (1993). Reliability coefficients (Cronbach) of the four dimensions of the inventory (learning styles) are between 0.73 and 0.83. In this study, the Turkish version of this inventory was used to identify the learning style of students as an assimilator, diverger, converger or accommodator.

A personal information form was also added to that Inventory to gather demographic information about the students such as previous geometry grades and past experience in the GeoGebra program.

Aşkar and Akkoyunlu (1993)' adapted LSI which consists of 12 items, each with four options (asking individuals to rank the four learning styles that best describe their own). Sample items from this inventory and explanations on scoring were given at Appendix 1.

2.3.2 Problem-solving rubric for task given at GeoGebra

Regarding the reality of students using various strategies, various problem-solving rubrics were developed by the researchers for observed strategies (Table 1). Such rubrics were used to assess student performance when solving this specific given problem at GeoGebra and to assign their grades. Student grades were finalised by confirming with two specialists in that area, and consistencies between 80 and 90% were observed in each.

2.3.3 Tobii studio

Eye-tracking data was obtained through Tobii Studio (eye-tracking software with measures such as 'Fixation Duration', 'Fixation Count' and 'Time to First Mouse Click').

After obtaining the permissions from the Middle East Technical University [METU] Human–Computer Interaction Laboratory [HCIL], we used the eye-tracking device available there. The hardware (eye-tracking device- Tobii T120) and software (Tobii Studio) required for our study were present in the laboratory."

There was a single eye tracker (Tobii 120) which collects data on where on the screen the user looks, how long a glance is, how many times the user looks at a certain point on the screen and all the eye movements of the subject during the experiment is also monitored (Fig. 3). The test computer which is used by participants that attached to the eye tracker is coupled to another computer in the control room (Fig. 4) which records the on-screen view of the user.

Tobii Studio is a software developed by the manufacturers of the eye tracker that transforms the information received from the reflectors having infrared detector cameras into visual and digital formats. It records these information

Table 1 One of the rubrics for a specific strategy used by students in that given task

Task solving process steps	Scoring
1. Select 'circle with center through point' tool from menu (e.g. centre is "A", any point on circle is "B")	1
2. Draw a circle with any radius	1
3. Select the line through two points' tool from the menu	1
4. Draw a line through centre (e.g. A) and any point (e.g. B) on the circle	1
5. Select 'intersect two objects' tool from the menu to draw intersection point (e.g. 'D') of circle and line	1
6. Select 'slider' tool from the menu to slide chosen point (e.g. 'a') on any semicircle	1
7. Select 'Rotate Object around Point (e.g. B) by Angle' from 'Reflect Object (e.g. B^I) about Line' tool	1
8. Select 'Polygon' to draw a triangle BB^ID on semicircle	1
9. Select 'Angle' tool from the menu to find that angle B^I is 90 degrees	1
10. Slide point 'a' on semicircle to prove that angle B^I is always 90 degrees	1

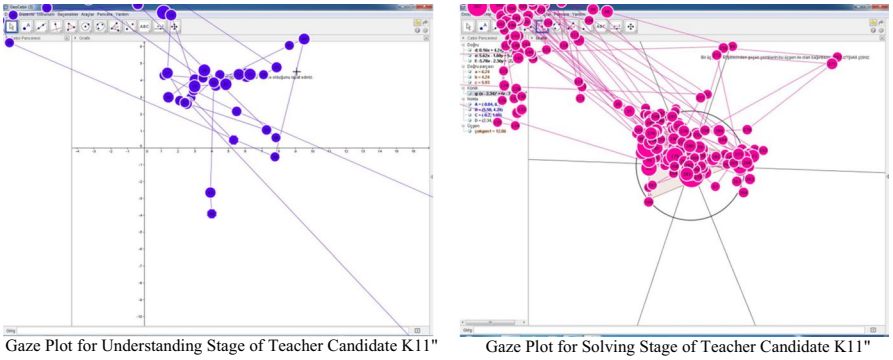

Fig. 3 Screen captures obtained from Tobii Studio belonging to the participants

Fig. 4 The photo of control room of HCIL in the METU

and later provides various tools for analysing the data saved (Human Computer Interaction Laboratory [HCIL], 2023).

2.3.4 Video recording and checklist of students

Each student was recorded while problem solving in the laboratory in two ways: i) through captured screen videos by Tobii 120 and ii) researchers' outer camera recordings while they were allowed to observe themselves loudly. This helped to understand the intent of their behaviours more clearly. Video recording checklists for each were used to detail information from participants. These data were combined with eye-tracking statistics simultaneously. Video recording checklist for qualitatively analyzing problem solving strategies of each student is given at Appendix 2.

2.4 Data analysis

Data analysis was conducted quantitatively using Tobii Studio data, and qualitatively on Tobii Studio video screen recordings and outer video recordings of observed students. Tobii Studio gives quantitative eye-tracking statistics such as fixation durations, fixation counts and 'time to first mouse click' at each scene. All screen shots were converted to a video format. These video records were downloaded in.avi format for further qualitative analysis. In this study, video recordings for each scene were combined with eye-tracking statistical results. Together, these data helped to build a comprehensive picture of participants' behaviours.

2.4.1 Quantitative data analysis

Two scenes were created using Tobii Studio to investigate students while they were solving the problem. One was a reading scene and the other was a solving scene.

The reading scene covered the time from reading the question on the GeoGebra interface until any action was started to solve the problem.

The duration of the solving scene began with the act of selecting the tool(s) on GeoGebra's menu and ended with the completion of problem solving.

Area of ınterest (AOI) Areas of the screen that were determined and defined by the researchers. More than one area could be determined per screen (Fig. 5).

The scenes were divided into areas of interest in Tobii Studio to take eye-tracking statistics of participants from the specified areas during their problem solving. Researchers could observe users' eye movements and use these to track students' actions in their problem solving. In this study, researchers divided scenes into five areas of interest: question, menu, input, algebra and graphic. An example of

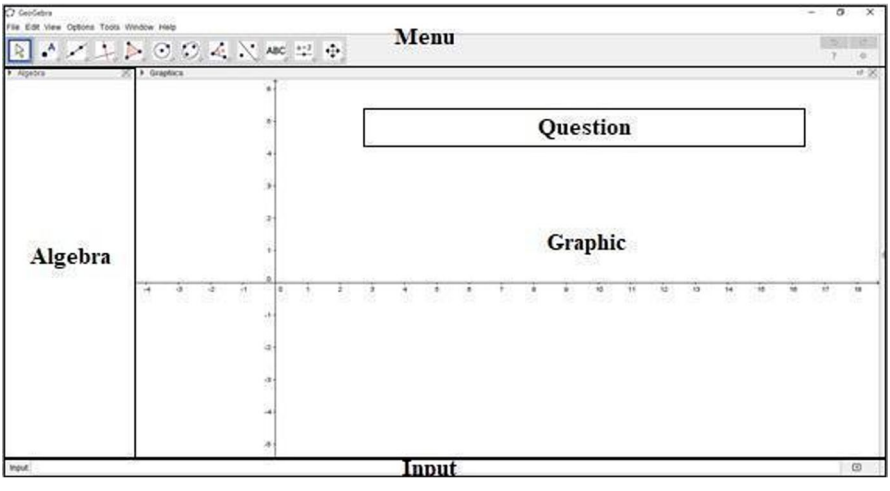

Fig. 5 An example screenshot for AOI on the GeoGebra interface

the GeoGebra interface based on a screen shot with the divided areas of interest is shown in Fig. 5.

The activities that participants were requested to undertake and the statistics that researchers could derive from these areas of interest are defined below and demonstrated in Fig. 6.

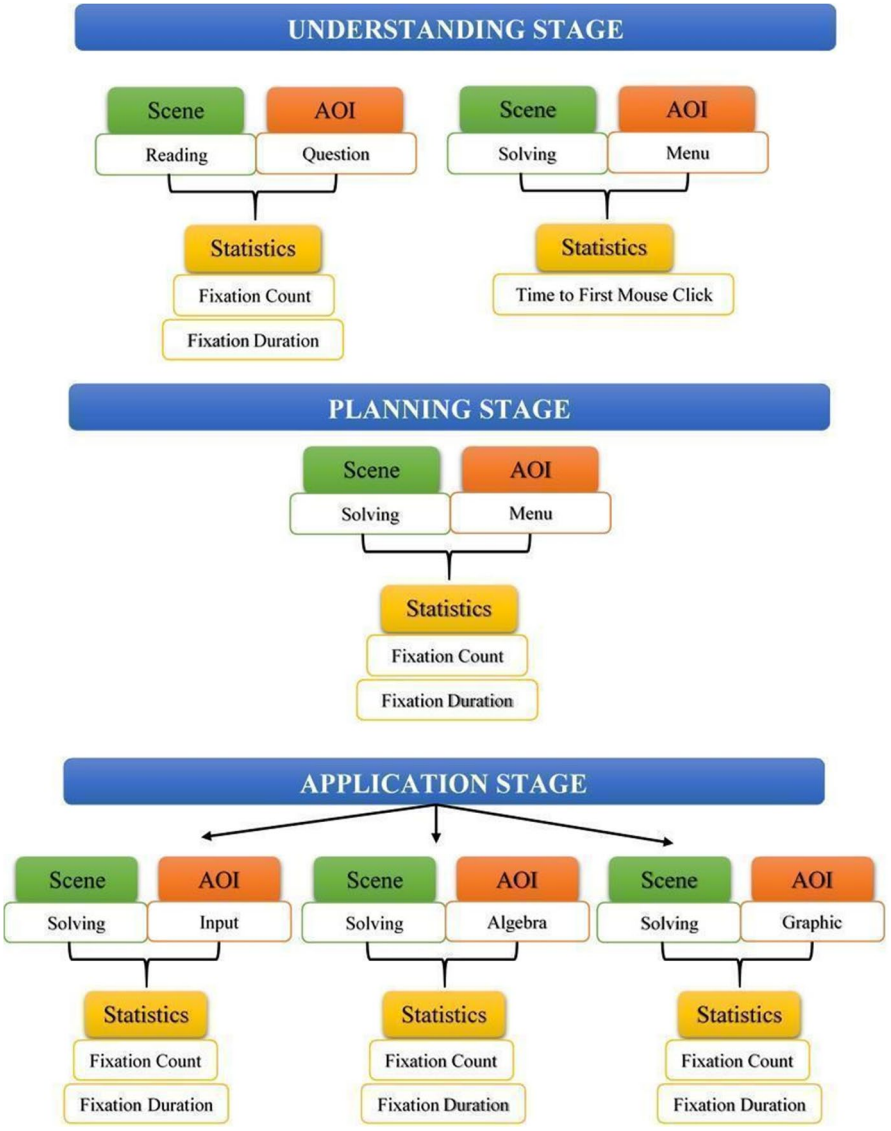

Fig. 6 Related scene and AOI combinations for each Polya's (2004) stage

Question Participants were required to read the question. We used fixation count and fixation duration statistics from this area to investigate participants' problem-solving skills related to their understanding stage at the reading scene.

Menu In this area of interest, students were required to select tools from the GeoGebra menu to solve the problem. We used fixation count and fixation duration statistics from that area to investigate participants' problem-solving skills at the solving scene related to the planning stage. At this stage, 'time to first mouse click' statistics obtained from the menu area were also used at the solving scene in the understanding stage.

Input Students were required to enter values and see the consequent solution. We used fixation count and fixation duration statistics from this area to investigate participants' problem-solving skills related to the application stage as well as in the graphic and algebra areas at the solving scene. The Input Bar allows participants to directly create and redefine mathematical objects in the Algebra View by entering or modifying their algebraic representations (e.g. values, coordinates, equations).

Algebra Students were required to solve problems by entering algebraic expressions into the integrated input field or the input bar. The graphical representation of this is automatically displayed in the Graphics View. We used fixation count and fixation duration statistics from this area to investigate participants' problem-solving skills related to the application stage as well as in graphic and input areas at the solving scene.

Graphic Participants were required to solve the problem by drawing graphs. For this area, students could make direct changes in the graphic area by switching the intersection area with their mouse, without using any tools from the menu. We used fixation count and fixation duration statistics from this area to investigate participants' problem-solving skills related to the application stage as well as in algebra and input areas at the solving scenes.

For this part, it was also convenient to describe the metrics that were used in statistics in Tobii Studio (2022) for the data obtained.

Fixations Analysis of fixations typically involves assessing their location (i.e., the object being fixated on) and measuring their frequency (fixation count) and duration (fixation duration) within a region of interest.

Fixation duration Duration of each individual fixation within an AOI measured in seconds.

Fixation count Number of times the participant fixated on an AOI.

Time to first fixation duration The time from the start of the media display until the test participant fixates on the AOI or AOI group for the first time, measured in

seconds. This metric measured how long it took before a participant fixated on an active AOI for the first time.

Sum Multiplication of fixation count and fixation duration.

All created scenes, AOIs and eye-tracking statistics used in each combined scene and AOI for selected problem-solving Polya' stages (2004) are summarised in Fig. 6.

2.4.2 Qualitative data analysis

Qualitative data that contained participants' real screen tracking information from the Tobii video recordings and students' outer camera video recordings underwent document analysis. These data were combined with eye-tracking statistics.

3 Findings

The results of the eye-tracking statistics taken during students' problem solving on GeoGebra are explained below, following the three problem-solving stages: i) understanding, ii) planning and iii) application. Such data were supported with results of qualitative data, i.e., observations from video recordings (Appendix 2), whenever more detail was required.

To investigate students' general behaviours in understanding, planning and application stages, eye-tracking data were mainly obtained from the following screens at specified areas of interest such as: solving scenes at menu interest of area (solving-menu), reading scene at question interest of area (reading-question), solving scene at input of area (solving-input), solving scene at algebra interest of area (solving-algebra) and solving scene at graphic interest of area (solving-graphic) as summarised in Fig. 6.

3.1 Eye-tracking analysis results of students' problem-solving skills related to understanding stage

The results of the eye-tracking analysis, presented as statistics of students' problem-solving skills in the understanding stage, are indicated in Tables 2 and 3 with student characteristics such as learning style, previous overall grades from geometry and grades on the task given during this study.

Considering the fixation time alongside the data obtained from video recordings, observation forms and student grades provides a good indication of student behaviours in the understanding stage.

As summarised in Fig. 6, to investigate students' problem-solving behaviours and skills at the understanding stage, two focus areas were determined, and the results of eye-tracking analysis on those focus areas were presented as the i) reading-question (Table 2) and ii) solving-menu (Table 3).

Table 2 Eye-tracking analysis results of students' problem-solving skills related to understanding stage: reading-question

Scene	Area of interest	Participant	Learning style	Task grade	Eye-tracking statistics		
					Fixation count (*N*)	Fixation duration (M)	Sum*
Reading	Question	K11	Converger	10	13	0.33	4.35
		K10	Converger	8	8	0.52	4.18
		K1	Converger	8	11	0.37	4.09
		K6	Assimilator	10	15	0.25	3.72
		K12	Converger	8	10	0.29	2.91
		K2	Assimilator	2	1	0.18	0.18
		K8	Assimilator	2	1	0.17	0.17
		K9	Converger	8	-	-	-

*Total fixation duration

Table 3 Eye-tracking analysis results of student problem-solving skills related to the understanding stage: solving-menu

Scene	Area of interest	Participant	Learning style	Task grade	Eye-tracking statistics		
					Time to first mouse click (s)		
					N	M	Sum*
Solving	Menu	K11	Converger	10	1	38.27	38.27
		K6	Assimilator	10	1	17.38	17.38
		K10	Converger	8	1	16.53	16.53
		K8	Assimilator	2	1	12.24	12.24
		K9	Converger	8	1	11.96	11.96
		K12	Converger	8	1	10.6	10.6
		K2	Assimilator	2	1	10.54	10.54
		K1	Converger	8	1	9.3	9.3

*Total fixation duration

For the reading-question, the total time students devoted to reading the question and the number of clicks were investigated (Table 2).

Table 2 shows student intensity in focusing on the question. Looking at the fixation duration and number of total gazes (fixation counts) indicates that the most successful students (K6 and K11) had the highest fixation count. That indicates they read the question many times in order to understand it. Such action to understand the question could play an important role in the ultimate problem-solving success. K9 was observed tracking the question box to other areas of the screen that caused a loss of data. It is clear from Table 2 that the two unsuccessful

students (K2 and K8) showed the smallest number of fixation counts and fixation duration, implying a lack of attention to the question. Here, fixation count seems to be an indicator of high grades from the task whereas no such kind of indication exists with learning styles.

In light of the data in Table 2, it is believed that aside from progress at the menu area, student progress in reading the question area seems to be one the most important actions for success. To summarise all these behaviours and make a general conclusion about the situation in the understanding stage based on the learning styles, it was decided to investigate the case from the direction of 'SUM' in eye-tracking statistics that related to the reading-question area. It is believed that results reflecting both fixation count and duration could explain student progress in the reading phase in understanding the problem. In that sense, looking at Table 2.

- Two most successful students had the highest fixation counts in reading the question area.
- No obvious pattern of relation was observed between student success and time spent solving in the understanding stage, although two successful students devoted a lot of time here.
- Converger students (K11, K10, K1) generally had most of the highest sums and grades (4.3, 4.18 and 4.09) except K12.
- Only one assimilator (K6) had the highest grade (10) and one of the highest sums. This is an exception and could be explained on the basis of his/her reading the question many times and giving attention to understand the problem (fixed counts).
- The two unsuccessful assimilator students (K2 and K8) had the least sums of fixed counts and durations (0.18 and 0.17), and they had devoted the least time to reading the question.

For the solving-menu, eye-tracking statistics of 'time to first click' are given in Table 3.

These data give information about the duration from when students started reading the question until the first mouse click on that screen. This could be a good indication of a student's readiness to start solving the problem in GeoGebra as well as his/her decision to choose appropriate menu tools through that action. When we look at Table 3, this could give some insight when considered together with the grades from the task. K11, a student with a converger learning style who had the highest grade for that question, devoted the most time to thinking before starting to solve the question. Observations of video recordings also indicated that this student read the question many times aloud and gazed at certain points on the screen before any action was taken. Another converger, K1, had the best background in geometry and of all students, devoted the least time before any click. A clear point that emerged from the data is that assimilators typically failed (except K6), while all convergers were successful with the question. It may be noted from Table 3 that all assimilators, and the convergers who felt the problem was difficult, devoted longer times before taking any action in the menu area.

3.2 Eye-tracking analysis results of student problem-solving skills related to the planning stage

Eye-tracking analysis results of students that indicate problem-solving behaviours in the planning stage are shown in Table 4. To investigate the problem-solving behaviours/skills of students at this stage, their behaviours were analysed in terms of statistics such as the duration (fixation duration) and number of times they focused (fixation count) on the related area in the solving-menu scene (Fig. 6). The reason for the consideration of such statistics is to have the knowledge about the menu and tools that students planned and preferred to use when they start to solve the problem. The data obtained from video records, observation forms and students' grades were also important in clarifying the nature of the planning stage.

In Table 4, student K9 had the highest fixation count ($N = 103$). As also noted in the observation of students, a lack of understanding about the question, not finding a correct strategy quickly and having difficulty deciding how to use appropriate menu tools could be the main reasons behind such data. Video recordings clearly showed this student's struggle with and resolution of the challenges, and this could be a key point in getting a high grade for that question. In contrast, K12, who showed the lowest fixation count and fixation duration in the solving-menu area, was observed in video recordings reading the question rapidly and choosing the correct menu tools in a short time.

General analysis of student behaviours for the planning stage (solving-menu scene) are below:

- Students who felt the question was difficult spent a longer duration of time in that area.
- Students with the same learning style did not show similar performance in the menu area. Some students had difficulty finding the correct tool to solve the prob-

Table 4 Eye-tracking analysis results of student problem-solving skills at the planning stage in solving-menu

Scene	Area of interest	Participant	Learning style	Task grade	Eye-tracking statistics		
					Fixation count (*N*)	Fixation duration (M)	Sum*
Solving	Menu	K9	Converger	8	103	0.4	41.5
		K2	Assimilator	2	45	0.57	25.72
		K11	Converger	10	22	0.67	14.82
		K1	Converger	8	25	0.3	7.45
		K6	Assimilator	10	20	0.26	5.21
		K8	Assimilator	2	16	0.31	4.91
		K10	Converger	8	8	0.49	3.95
		K12	Converger	8	1	0.25	0.25

*Total fixation duration

lem while some of them already knew the standard menu structure and found it easily (e.g. K9 and K12).

3.3 Eye-tracking analysis results of student problem-solving skills related to the application stage

To investigate student problem-solving behaviours and skills at the application stage, three intention areas were determined, as presented in Fig. 6. Those screens were selected because students used them during the problem-solving process. Eye-tracking results related to this stage were presented as i) solving-input (Table 5), ii) solving-algebra (Table 6) and iii) solving-graphic (Table 7).

For the solving-input area of interest, students tried to enter different variables to see the resulting graph in the graphic area. The results presented fixation counts and fixation durations of the students at the solving-input area when they had entered values and were using formulae there. Statistics from that area were intended to inform our understanding of the ways in which students solved the problem. Students were not required to use this area, so this provided good insight into their choice and approaches to solving the problem.

As shown in Table 5, only three of the students (K2, K9 and K11) used that area. K2 was one of the failing students who did not understand the problem. Video recordings and observations of students at that screen indicated that this student was not interested in solving the problem, similar to K8. However, the fixation counts and durations of converger K11 ($N=12$ with 3.18) indicated a strong effort to reach a solution in that area. This was also clear through video recordings and observation results for this student, which showed he/she had entered different variables and formulae. Both convergers favoured the use of formulae and their strength in formula writing was obvious in the analysis.

Table 5 Eye-tracking analysis results of student problem-solving skills related to the application stage: solving-input

Scene	Area of interest	Participant	Learning style	Grade	Eye-tracking statistics		
					Fixation count (*N*)	Fixation duration (M)	Sum*
Solving	Input	K11	Converger	10	12	0.27	3.18
		K2	Assimilator	2	2	0.27	0.53
		K9	Converger	8	1	0.1	0.1
		K1	Converger	8	-	-	-
		K6	Assimilator	10	-	-	-
		K8	Assimilator	2	-	-	-
		K10	Converger	8	-	-	-
		K12	Converger	8	-	-	-

*Total fixation duration

Table 6 Eye-tracking analysis results of student problem-solving skills related to application stage-solving algebra

Scene	Area of interest	Participant	Learning style	Task grade	Eye-tracking statistics		
					Fixation count (*N*)	Fixation duration (M)	Sum*
Solving	Algebra	K11	Converger	10	50	0.37	18.44
		K2	Assimilator	2	38	0.33	12.67
		K9	Converger	8	40	0.31	12.39
		K6	Assimilator	10	38	0.23	8.69
		K1	Converger	8	18	0.24	4.35
		K10	Converger	8	6	0.3	1.8
		K12	Converger	8	6	0.19	1.13
		K8	Assimilator	2	4	0.24	0.97

*Total fixation duration

Table 7 Eye-tracking analysis results of student problem-solving skills related to application stage: solving-graphic

Scene	Area of interest	Participant	Learning style	Task grade	Eye-tracking statistics		
					Fixation count (*N*)	Fixation duration (M)	Sum*
Solving	Graphic	K9	Converger	8	694	0.56	386.94
		K1	Converger	8	1088	0.26	282.62
		K11	Converger	10	800	0.34	269.72
		K8	Assimilator	2	237	0.55	129.31
		K2	Assimilator	2	249	0.42	105.11
		K6	Assimilator	10	313	0.32	99.38
		K12	Converger	8	212	0.29	62.21
		K10	Converger	8	76	0.77	58.72

*Total fixation duration

- Students generally didn't prefer to use the input area.
- It is noteworthy that only three students opted to use that area, two of which were convergers.
- Examining student behaviours in terms of success in the application stage through their fixation durations and counts in the solving-input area were more meaningful when interpreted in parallel with qualitative data such as their outer video recordings.

Another indicating factor was student behaviours in the application stage, measured through their fixation durations and counts in the solving-algebra area as presented in Table 6.

The results shown in Table 6 indicate that K11, a converger, had the highest fixation count in this area ($N=50$). Another converger, K9, also had a high count there ($N=40$). This was not surprising, since those students were also the ones who used the solving-input menu the most. Apart from K2, who paid little attention to solving the problem, the high Sums of K11 (18.44s) and K9 (12.39s) were an indication of the emphasis they placed on solving the question, which seems to have been ultimately reflected by high grades (10 and 8, respectively).

- The only students who used the input area (K11, K2 and K9) were also the ones with the highest Sums in Table 6. Those students devoted long times and more effort in the algebra area except K2, who had been observed as having a kind of lazy and careless time here.
- As seen in this table, high fixation durations and counts were not always indicators of high grades, as was the case for the short durations of K10 and K12.
- Examining student behaviours in the application stage through their fixation durations and counts in the solving-algebra area were more meaningful when interpreted in parallel with qualitative data such as their outer video recordings.

To understand student behaviours at the application stage, investigation of their fixation counts and durations at the solving-graphic area was also important (Table 7).

Table 7 shows that K1 had the highest fixation count (he/she looked most intensely at that area), and it was clear that he/she spent a longer amount of time in that area than other students. From the observations of this student, the reason for this was his/her attempts to make proofs for his/her solution. K10 had the smallest fixation count and fixation duration in the graphic area. This student was observed quickly noting two important points: i) what is given and ii) what is required for the problem. Examining student behaviours in the application stage through their fixation durations and counts in the solving-graphics area were more meaningful when interpreted in parallel with qualitative data such as their outer video recordings.

3.4 Student behaviours based on their general characteristics

Before a general conclusion on the results of eye-tracking statistics, we summarise the most obvious behaviour of students in each problem-solving stage based on their characteristics from the quantitative and qualitative data.

Starting with converger students, student K1 obtained a high grade (8). We looked through his/her behaviours at each step of problem solving, and some patterns of behaviour were investigated. Compared to other students, this student spent the least amount of time on the understanding stage (9.3s; Table 3) in starting his/her first action. In light of the student's score, this could be an indication of being conscious of his/her actions in addition to this being one of the students who devoted

the longest times and gazed frequently at the reading-question area (Table 2; $N=11$ with 4.09s). This indicates an appropriate effort made by K1 in the understanding stage. Devoting above-average time and gaze ($N=8$, 7.45s; Table 4), K1 also showed some effort in the planning stage, although K2 and K11 had higher values there. Additionally, K1 was observed as devoting more time and focus to the solving-graphics area ($N=1088$, 282.62s; Table 7). This could be a good indication of effort devoted in the application stage with more concentration on visual procedures.

Data from another converger student, K9, showed him/her waiting before the first action (11.96s; Table 3) as a sign of emphasis on the understanding stage although the time spent reading the question was so long. His/her high values indicated a great effort in the planning stage with many gazes and the longest time spent there ($N=103$, 41.5min; Table 4). At the application stage, a considerable effort was again observed through the solving-algebra and solving-graphic areas as well as being one of the top students in the solving-algebra area ($N=50$, 18.44s; Table 6).

Another converger student, K11, had the highest grade for the question (10) and was observed taking the most time before starting any action (Fixation duration of 38.27s; Table 3), and devoting the most gazes and time among the students ($N=13$, 4.35s; Table 2) to reading the question in the understanding stage. In observation notes, it was clear that the planning of his/her own actions at using menus was overt. He/she spent considerable time and gazed frequently at the solving-menu area when choosing the correct tools to use ($N=22$, 14.82s; Table 4) at the planning stage. At the application stage, K11 was one of three students to use the solving-input area. The student also came first in the solving-algebra (Table 6) and second in solving-graphics (Table 7) areas when compared to all students at the application stage. Related data for the two other convergers (K10 and K12) also showed similar patterns to each other and were observed devoting slightly above-average effort in each step.

K2 and K8, students with the assimilator learning style, were seen through observation records as finding the question very difficult. They did not attempt to solve it seriously. The data in Table 2 are impressive in this respect because it clearly shows how these students devoted the least time and least gazes to reading the question (K2: $N=1$, 0.18s; K8: $N=1$, 0.17s). In contrast, K6, another assimilator, was awarded one of the top grades (10) and was observed giving sufficient emphasis at all problem-solving stages.

4 Conclusion and discussion

Results of this study indicated that students who devoted longer time to read the question and had a high number of fixations in the reading-question area at the understanding stage were the ones who got the highest grades also. This result was in accordance with Sajka and Rosiek (2015) and Dobias et al.. (2017), in which they stated that high duration time and high fixation counts in reading the question area generally contributed to success. Chettaoui et al. (2023) also suggest that "combining eye-gaze tracking with learning traces and behaviour attributes may support an accurate prediction of students' learning performance".

Behaviors of students in solving-menu at understanding stage mainly related their decision and first action to start to use the appropriate menu. Here, students who get the highest grades were observed as the ones who devoted longest time before their first action to use the menu tool in the GeoGebra interface. This was contradictory to results of Tóthová et al.(2021) as no obvious pattern of relation was declared with students' time to their first mouse click and their grades from the task.

Students' behaviours at the application stage were investigated in terms of eye statistics and behaviours in input, algebra and graphic areas respectively in our study. Students did not prefer to use the input area. Moreover, they had longer fixation counts in the graphic area than algebra as also declared in the study of Andra et al. (2015).

Students not preferring to use solve-input area where they could enter values and calculate, could be explained on the basis of Rahmah et al. (2022) conclusion in their study. They stated that students that had assimilator and converger learning styles generally solve problems correctly from the direct action, since there would be errors in putting variables in solving steps.

In our study, some common patterns were observed based on students' learning styles. It had been indicated very obviously in scope of this study that students with a converger type of learning style were all more successful in understanding the problem, planning to solve it, and in realizing their plan through application, than assimilator types of students. Convergers made a great deal of effort at the understanding stage, planned their way to solution in a shorter time and applied their plan precisely. On the other hand, when those students did not understand the question, they overcame this through good planning and taking care in applying that plan. Another interesting point in convergers' behaviour was the way in which they devoted long times in the application stage in case of having difficulty in solving the problem. This situation could not be defined as an indication of failure, and it should be kept in mind that these students indeed devoted that time to backward chaining to proof their solution as seen in their video recordings. Results of this study show commonly used successful behaviours in accordance with the results of Hegarty et al., (1992, 1995) and Hu et al. (2017) which emphasized the effectiveness of focusing on planning application steps.

According to the results of our study, all students who failed to solve the problems were assimilators. Rohmanawati et al. (2021) explained and generalized the assimilators' characteristics as their tendency to describe the question and to search known solutions to similar problems.

There was a very clear indication regarding assimilators in which, although they all shared the same background from their geometry grades, they differed in terms of their grades and their behaviours while solving the problem. Differing from the other two assimilators, Student K6 obtained one of the highest grades for the question, and their behaviours at each problem-solving stage were very similar to other converger students. Another common pattern noted in all students was that failure was particularly the result of not understanding the problem.

It is crucial to note that all comments are provided within the scope of the findings of this study. Since the study is a case study, generalisations have been avoided. From the findings, it could generally be concluded that there was no clear indication

for the differentiation of behaviours, based on learning styles. In contrast, there were clear indications of differentiation of behaviours when based on students' success in terms of their task grades. It could be concluded that students' grades were more meaningful for interpreting their problem-solving behaviours that arose in the context of eye statistics but learning styles could also provide clues in this regard.

In conclusion, when teaching geometrical concepts, knowledge of students' main problem-solving behaviours and their individual characteristics (such as their learning style) is important for designing effective instruction. This is particularly important in instructional design considerations that focus on supplying efficient support and feedback to students.

5 Limitations and suggestions

This study has several limitations. Firstly, the study relied on data from a relatively small sample size. However, it was adequate to thoroughly explore the behaviours of the participating students while solving a specific maths problem in GeoGebra within the confines of this study. As noted by Boddy (2016), case studies involving even a single case can be significant and provide valuable insights for future research, as well as shedding light on previously unexplored phenomena. Additionally, it is important to mention that the participants in this study were exclusively female. Future studies could include participants of different genders. Furthermore, our study focused only on two learning styles: assimilators and convergers. Similar studies could be conducted to examine other two learning styles as well. Another limitation was the exclusion of the revision stage in problem solving to prevent complications in eye-tracking analysis due to frequent back-and-forth checking. It is recommended to devise alternative designs that incorporate the revision stage while avoiding overlapping areas of interest in similar studies. Moreover, the study was limited to university students from the Department of Education in a private university. The observed behaviour of students, which centered on problem-solving in algebra, should be interpreted cautiously when considering the applicability of the study's results to other populations and disciplines in future research.

Regarding limitations of the tool itself, using eye tracking in understanding students' problem solving skills in geometry, Schindler and Lilienthal (2019) suggest that "a certain ambiguity will remain in the interpretation of data even in a subdomain-specific interpretation". Researchers acknowledge that eye tracking technology has limitations, as visual attention to particular locations on a task sheet can be related to different mental processes. However, they also assert that this tool can provide insights into what students are paying attention to and can be used to give feedback accordingly.

Eye tracking is a method to observe students' mathematical task solving process and crucial for understanding how task design can affect students' mathematical

reasoning (Norqvist et al., 2019). In this study only one geometric task was given to participants to understand how to prove the theorem step by step. The different tasks requiring geometrical generalisation can be given.

Appendix 1. Adapted Kolb learning style inventory

Aşkar and Akkoyunlu (1993) adapted Kolb Learning Styles Inventory consists of 12 items, each with four options, asking individuals to rank the four learning styles that best describe their own. Sample items from this inventory and explanations on scoring were given in the below;

Example item:

a) When learning a new topic, which of the following four options best describes you?
b) Learning based on emotions and sensory experiences
c) Observing and reaching conclusions based on my observations
d) Learning abstract ideas and theories"
e) Doing something

Scoring:
Each option represents 4 learning styles. (Rank from 4 to1)

1. Option 1: Concrete Experience (CE)
2. Option 2: Reflective Observation (RO)
3. Option 3: Abstract Conceptualisation (AC)
4. Option 4: Active Experience (AE)

A score between 12 and 48 is obtained as a result of the points given by the respondents to each option. The next step is to obtain the combined scores. If it is,

AC-CE: Abstract Conceptualisation-: Concrete Experience.

AE-RO: Active Experience- Reflective Observation is obtained as.

The scores obtained from these totals also range from -36 to + 36. The positive score obtained in AC-CE indicates that learning is abstract, negative score indicates that learning is concrete. Likewise, positive and negative scores obtained in AE-RO indicate whether learning is active or reflective. The intersection point of two scores, obtained by combining (AC-CE, AE-RO) as shown in Fig. 7 provides the most suitable learning style for the individual.

The Table 8 shows the scores participants received from the Kolb Learning Styles Inventory and the corresponding learning styles associated with the scores in the below.

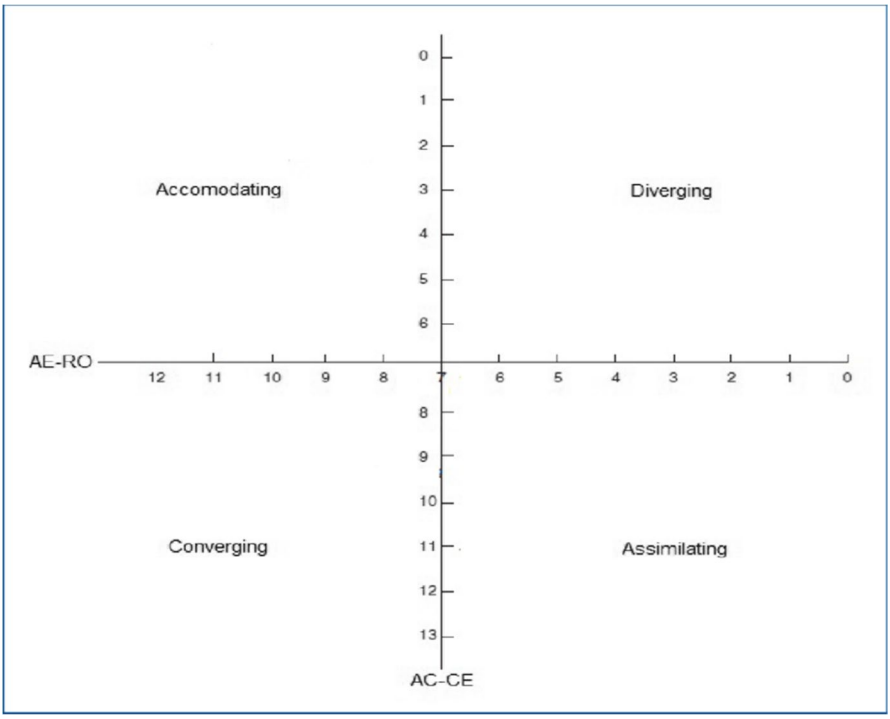

Fig. 7 Kolb Learning Style Type Grid (Kolb & Kolb, 2005a)

Table 8 Learning styles of participants

Participants	CE	RO	AC	AE	AC-CE	AE-RO	Learning style
K1	19	23	33	45	14	22	Converger
K2	21	29	38	32	17	3	Assimilator
K6	23	35	34	28	11	-7	Assimilator
K8	26	27	43	24	17	-3	Assimilator
K9	15	35	27	44	12	9	Converger
KP10	24	21	42	33	18	12	Converger
K11	23	25	28	44	5	19	Converger
K12	24	24	33	39	9	15	Converger

Appendix 2. Video recording checklist of students

Understanding stage	**Not done**	**Partially done**	**Done**
Did they understand all the words used in the problem?			
Did they grasp all the concepts mentioned in the problem?			
Did they comprehend what was expected of them?			
Could they express the problem in their own words?			
Could they prepare a rough sketch or diagram for solving the problem?			
Is the information provided sufficient to solve the problem?			
Planning stage	**Not done**	**Partially done**	**Done**
Could they establish relationships among the unknowns and the given information?			
Have they encountered a similar problem before?			
Do they know an appropriate theorem or model that can be used to solve the problem?			
Can they express the problem differently?			
Have they considered solving the problem in a different way, and have they determined the necessary data?			
Have they used all the data given in the problem?			
Did they anticipate that the angle subtended by the diameter of the circle should be 90 degrees?			
Was a triangle drawn inside the semicircle?			
Application stage	**Not done**	**Partially done**	**Done**
Was the line segment AB* drawn?			
Were circles drawn with centers at points A* and B*?			
Was a point C* created on the semicircle?			
Was it ensured that this point C*, when dragged, remained on the circle?			
Was the triangle ABC* created?			
Were the interior angles of triangle ABC* measured to verify the result?			
Was it confirmed that the line drawn from the center of the semicircle (O*) to point C (line OC*) is indeed the radius?			

*The points, lines, line segments, triangles, and similar geometric shapes represented in the form may vary in their names according to the students' responses

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Data availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics This manuscript has not been published or presented elsewhere in part or in entirety and is not under consideration by another journal. We have read and understood your journal's policies, and we believe that neither the manuscript nor the study violates any of these.

Consent to participate/consent for publication Informed consent was obtained from participants before the study. The participants were volunteers who provided written informed consent. They were informed that they had the right to withdraw from the study at any time without penalty. Confidentiality was ensured by using numbers instead of names in the research data base. Data were only used for research purposes.

Competing interest The authors have no relevant financial or non-financial interests to disclose.

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