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ARTICLE

Selective generalized travelling salesman problem

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ABSTRACT

This paper introduces the Selective Generalized Traveling Salesman Problem (SGTSP). In SGTSP, the goal is to determine the maximum profitable tour within the given threshold of the tour's duration, which consists of a subset of clusters and a subset of nodes in each cluster visited on the tour. This problem is a combination of cluster and node selection and determining the shortest path between the selected nodes. We propose eight mixed integer programming (MIP) formulations for SGTSP. All of the given MIP formulations are completely new, which is one of the major novelties of the study. The performance of the proposed formulations is evaluated on a set of test instances by conducting 4608 experimental runs. Overall, 4138 out of 4608 (~90%) test instances were solved optimally by using all formulations.

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1. Introduction

Routing problems with profits are different from classical routing problems in that not all nodes (customers, cities etc.) need to be served. Each node has an associated profit, and the right set of nodes must be served to satisfy a certain number of side constraints while maximizing the profit gained from the visited nodes. In the field of routing problems with profits, the Travelling Salesman Problem (TSP) with Profits (TSPPs) indicates a set of problems in which a single mobilized entity performs the operation. Depending on the constraints and objective function considered in the problem, TSPPs are classified into three categories [1]: the Orienteering Problem (OP) (or the Selective TSP), the Prize-Collecting TSP (PCTSP), and the Profitable Tour Problem (PTP).

In the OP (or in the STSP), the goal is to determine which subset of nodes to visit and in which order such that the total collected profit is maximized and a given maximum total travel time is not exceeded [2]. This problem was first introduced by Tsiligirides [3]. For extensive and comprehensive surveys on the OP, the papers of Vansteenwegen et al. [4] and Gunawan et al. [5] are recommended.

In this paper, we address a generalization of the STSP problem family, a variant of the Generalized TSP (GTSP) that we call the Selective GTSP (SGTSP). In this problem, nodes are clustered into groups. Neither all clusters nor all nodes need to be visited, but only one node within each cluster may be visited. A profit is associated with each node and collected only if served. The aim is to maximize the total profit collected within a given time limit. The literature also studies a problem called the Generalized OP (GOP), in

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which each node is assigned a set of scores with respect to a set of attributes (for example, a tourist trip-designing problem that considers attributes such as natural beauty, historical significance, cultural and educational attractions, and business opportunities [6];). Wang et al. [7] considered the objective function of the GOP to be a non-linear function of these attributes, demonstrating that the generalization of the OP is done in terms of having multiple goals, and the objective function is a non-linear combination of the attribute scores. This type of GOP was first studied by Wang et al. [8]. Silberholz and Golden [9] and Pietz and Royset [10] have published recent papers as well. However, in our study, the SGTSP is a problem in which each node is associated with only one score (profit value) and this generalization is done in such a way that the TSP is generalized to the GTSP (see [11–17]). Interested readers can refer to the generalized minimum spanning tree problem ([18,19]) and generalized vehicle routing problem [20] which are related with GTSP. We consider only a single objective function with a linear combination of profits. In a recent study, Archetti et al. [21] studied the Set OP (SOP), a generalization of the OP in which nodes are grouped in clusters, a profit is associated with each cluster, and the route duration does not exceed a given time limit. The objective is to find the route that maximizes profit. The profit of a cluster is collected only if at least one node from the cluster is visited. They proposed a mathematical formulation of the problem and a matheuristic algorithm. Unlike in Archetti et al. [21], the objective function in this study is the sum of node profits visited on the route.

By studying this problem, the practical applications can be analysed and formulated as variants of the SGTSP. Examples of such applications are mainly related to the distribution of mass products, as in the case of a travelling salesman without enough time to visit all nodes. He knows the number of sales to each node and wants to maximize his total sales while keeping the travel limited to a specific unit of time [3]. In particular, consider the case where nodes belong to different clusters (country, city, local area, region etc.). Instead of having to serve all nodes belonging to the cluster in the SGTSP the carrier may choose to serve only at least one node from the cluster. By this way, within a certain time limit, the nodes that will maximize the benefit are visited and hence the total profit will be maximized. Other problems that might fit into this scope are Tourist Trip Design Problem, Routing of the Sales Representatives and Waste Collection. In the tourist trip design problem [22], it is often impossible to visit everything for the tourists visiting a city or region. Thus, they have to select what they believe to be the most valuable attractions. It is difficult to define a tour plan in the available time span in order to visit these attractions. In tourist applications the time required to visit a certain attraction point plays an important role in the selection of points. In the routing of field workers and sales representatives (e.g., sales representatives of food retailers or of the pharmaceutic industry), they have to visit the most promoting doctors or work places belonging to their own fields to fill their sales quota within the daily working time limit. Same as, in waste collection a vehicle or a fleet of vehicles visit a set of collection points within a time limit such that to maximize the total benefit.

The main contributions of the paper are in twofold. First, in the literature, two types of GTSP are studied ([23–25]). In the first type, only a single node may be visited, and in the second, at least one node is visited in each cluster on the tour. Generalized variations of OPs in which the nodes are categorized into clusters can be examined using the two headings above. In the literature there is only a study presented about the generalized variations of OP

[26] which is called as Set OP (SOP). Unlike the definition of SOP, profits are associated with nodes rather than clusters, to be able to examine the problem in two sub-headings mentioned above. Best of the authors' knowledge there is no study on the generalized variations of OP in which the profits are associated with nodes. In case the problem is defined in this way, the SGTSP can be examined in both categories where only a single node may be visited and at least one node is visited in each cluster on the tour. The differences between SGTSP and the problem addressed by Archetti's study are explained in detail in Section 4.1. The second contribution of this paper is that it proposes eight new mathematical formulations for the SGTSP using different kinds of auxiliary variables. The main difference between these formulations is the interpretation of the meaning of the auxiliary variables used when writing the subtour elimination constraints in different structures. Mathematical models are classified as being sequence-based (SB), time-based (TB), node-based (NB), and flow-based (FB) due to the meanings of the auxiliary variables used.

The remainder of this paper is organized as follows. In the second section, we give the formal definition of the SGTSP. Section 3 contains the new mixed integer programming formulations of the SGTSP. We show in Section 4 that the SGTSP and its formulations can be reduced to the special case of other problems. In Section 5, we do numerous computational experiments of the proposed MIP formulations on the benchmark instances. The paper concludes in Section 6 with some remarks and further suggestions.

2. Definition of the SGTSP

The SGTSP can be defined with the aid of a directed graph $G = (V, A)$ with vertices (nodes) $V = \{1, 2, ..., n\}$and connecting arcs $A = \{(i, j) | i, j \in V, i \neq j\}$. The vertices are grouped into k mutually exclusive and exhaustive vertex sets (clusters) in advance, such as$V = V_1 \cup V_2 \cup \cdots \cup V_k$ with $V_p \cap V_l = \emptyset, \forall p, l, p \neq l$. Connecting arcs are defined only between vertices belonging to different sets (clusters); there are no intra-set arcs. Each vertex $j \in V$has an associated nonnegative profit $s_j > 0$ that is assumed to be entirely additive and each defined arc $(i, j) \in A$ has a corresponding nonnegative travel time $t_{ij} > 0$. The starting/ending point is vertex 1, which is the element of $V_1 = \{1\}$ and fixed. Not all vertex sets (clusters) can be visited since the available time is limited to a given time budget T_{max}. The goal of the SGTSP is to determine a tour limited by T_{max} that visits exactly one vertex within some of the vertex sets in order to maximize the total collected profit. Figure 1 displays an example SGTSP defined on a directed graph with 10 vertices and five vertex sets. The lines illustrate a feasible tour that visits only four vertex sets.

Making use of the notation introduced above, the SGTSP can be formulated as an integer programming problem.

3. SGTSP mixed integer linear programming formulations

In this section we first present a general formulation for SGTSP, then specify the explicit forms of these models and proposed eight new formulations of the problem.

In SB formulations, the nodes are visited according to their order on a tour. TB formulations depend on the time distance between the nodes on a tour. Auxiliary variables are defined in a formulation to avoid sub-tours and capacity (time limit, load limit, etc.)

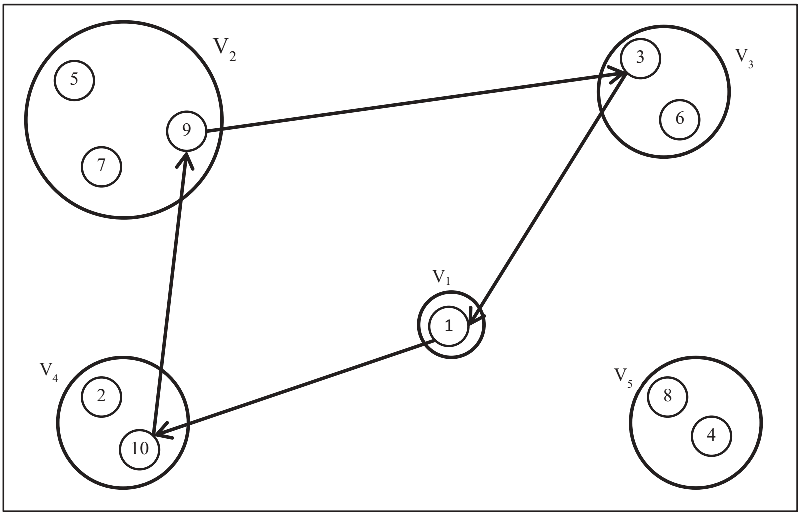

Figure 1. Example feasible SGTSP tour.

excess. If a formulation has additional auxiliary decision variables, it may be classified with respect to the new decision variables as being NB if the additional variables are relative to the nodes of the graph and FB if the new variables are relative to the arcs of the graph. Furthermore, the definition of auxiliary variables (both in NB and FB) is slightly different depending on where it is defined. If an auxiliary variable is defined between nodes, it is deemed to have a Node to Node (N_N) definition, and similarly, if an auxiliary variable is defined between clusters, it is considered to be Cluster to Cluster (C_C) defined. For instance, a NB auxiliary variable used in a formulation may be defined between nodes or between clusters. Likewise, a FB auxiliary variable used in a formulation may be defined between nodes or between clusters. Figure 2 shows the different formulations proposed in this paper, and gives their corresponding abbreviations.

All proposed formulations have $O(n^2)$ binary decision variables and constraints. All restrictions and objective functions in the formulations have linear relations. The parameters and the definition sets used in the formulations are given in the next sub-section.

3.1. A general formulation

We define the related sets, parameters, and decision variables as follows:

Symbols:

n is the number of nodes.

i and j are the indices of nodes $i, j = 1, 2, \ldots.., n$.

k is the number of clusters.

p and l are the indices of clusters $p, l = 1, 2, \ldots., k$.

Sets:

$A = \{(i,j) | i,j \in V, i \neq j\}$ is the set of arcs.

$V = \{1, 2, \ldots, n\}$is the set of nodes.

V_1 is the starting cluster including only the depot.

V_p is the set of cluster p.

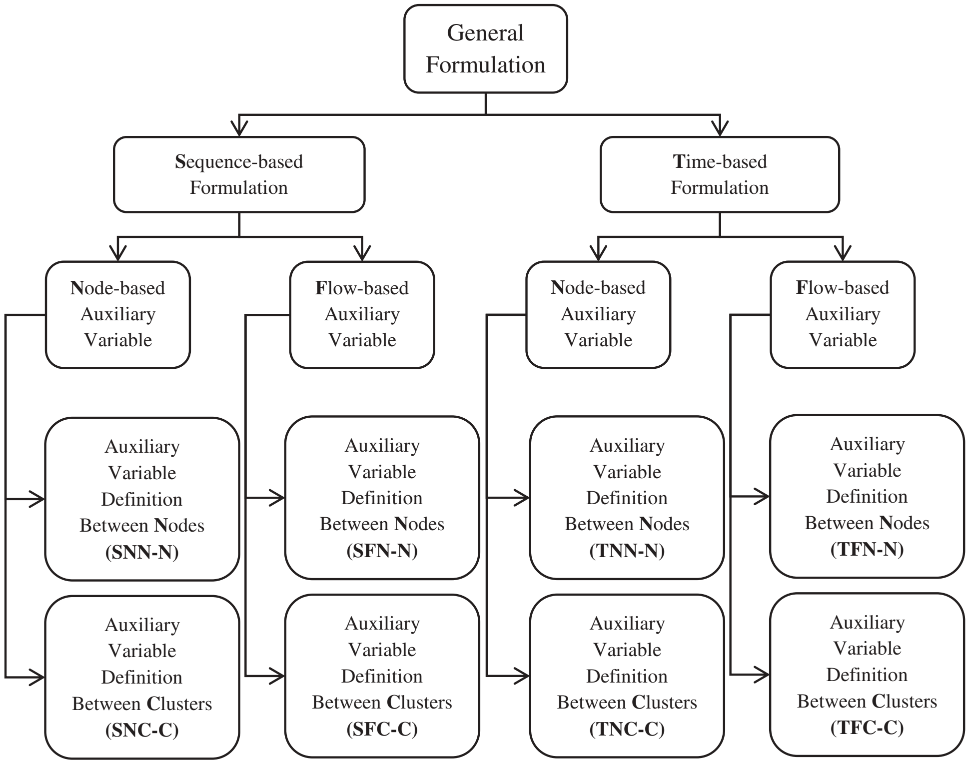

Figure 2. Mathematical formulation hierarchy.

Let V be partitioned into the mutually exclusive and non-empty subsets $V_1, V_2, V_3, \ldots, V_k$, each of which represents a cluster of nodes.

Parameters:

s_j is the profit (score, revenue, gain, etc.) of node j.

t_{ij} is the travel time between nodes i and j.

T_{max} is the maximum travel time.

Decision variables:

$$x_{ij} = \begin{cases} 1 \text{ if arc(i, j)is included in the tour of a vehicle} \\ 0 \text{ otherwise.} \end{cases}$$

With the above definitions, the general formulation for the SGTSP is given as:

$$\text{Maximize} \sum_{p=1}^{k} \sum_{i \in V_p} \sum_{j \in V \setminus (V_p \cup V_1)} s_j x_{ij} \tag{1}$$

The objective function (1) is to maximize the total collected profit.

$$\sum_{j=2}^{n} x_{1j} = 1 \tag{2}$$

Constraint (2) ensures that the tour starts from node 1.

$$\sum_{i=2}^{n} x_{i1} = 1 \tag{3}$$

Constraint (3) ensures that the tour ends at node 1.

$$\sum_{i \in V_p} \sum_{j \in V \setminus V_p} x_{ij} \leq 1 \, p = 2, \ \ldots, k \tag{4}$$

$$\sum_{i \in V \setminus V_p} \sum_{j \in V_p} x_{ij} \leq 1 \, p = 2, \ \ldots, k \tag{5}$$

Constraints (4) and (5) guarantee that each cluster is visited at most once.

$$\sum_{p=1}^{k} \sum_{i \in V_p} \sum_{j \in V \setminus V_p} t_{ij} x_{ij} \leq T_{max} \tag{6}$$

Constraint (6) ensures that the total travel time does not exceed the time limit T_{max}.

$$\sum_{i \notin V_p} \sum_{j \in V_p} x_{ij} - \sum_{i \notin V_p} \sum_{j \in V_p} x_{ji} = 0 \qquad p = 2, \ldots\ldots, k \tag{7}$$

$$\sum_{i \in V \setminus V_p} x_{ij} - \sum_{i \in V \setminus V_p} x_{ji} = 0 \qquad \forall j \in V_p \quad p = 2, \ \ldots, k \tag{8}$$

Constraints (7) and (8) are flow conservation constraints which can be used alternatively. Only one of these constraints is enough to allow the same node to be exited when a node is visited in a cluster.

and

$$\textit{Subtour elimination constraints} + \textit{Bounding constraints}$$

Model formulations of the SGTSP will differ from each other with respect to the subtour elimination and bounding constraints.

3.1.1. Sequence-based formulations

3.1.1.1. Formulation SNN-N: node-based auxiliary variable defined between nodes.

We propose NB MIP formulation for the SGTSP by defining a new auxiliary variable.

In addition to the decision variables defined above, we define the following auxiliary variable:

u_i is the order in which node i is visited after the depot, otherwise u_i is equal to zero.

With the above definition, the SNN-N formulation of the SGTSP is given as:

Objective function (1)

Subject to Constraints (2)–(8)

and

$$u_i - u_j + kx_{ij} + (k-2)x_{ji} \leq k - 1 \quad \forall i \in V_p \quad \forall j \in V \setminus (V_p \cup V_1) \quad p = 2, \ \ldots, k \tag{9}$$

$$u_i \geq x_{1i} + 2 \sum_{j \in V \setminus (V_p \cup V_1)} x_{ji} \quad \forall i \in V_p \quad p = 2, \ldots, k \tag{10}$$

$$u_i \leq (k-1)x_{i1} + (k-2) \sum_{j \in V \setminus (V_p \cup V_1)}^{n} x_{ij} - (k-3)x_{1i} \quad \forall i \in V_p \quad p = 2, \ldots, k \tag{11}$$

$$u_i \geq 0 \quad i = 2, \ldots, n \tag{12}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V_p, \quad \forall j \in V \setminus V_p, \quad p = 1, \ldots, k \tag{13}$$

Constraints (9) prevent subtours between clusters from using nodes. Constraints (10) and (11) are bounding constraints that limit the order of clusters. Constraints (12) are non-negativity constraints. Constraints (13) are binary constraints.

3.1.1.2. Formulation SNC-C: node-based auxiliary variable defined between clusters.

We propose another NB MIP formulation for the SGTSP by defining a new auxiliary variable. The difference with Formulation SNN-N is in the subtour elimination and bounding constraints, which prevent subtours between clusters.

In addition to the decision variables defined above, we define the following auxiliary variable:

u_p is the sequence from the depot to cluster p, otherwise u_p is equal to zero.

With the above definition, the SNC-C formulation of the SGTSP is given as:

Objective function (1)

Subject to Constraints (2)–(8)

and

$$u_p - u_l + k \sum_{i \in V_p} \sum_{j \in V_l} x_{ij} + (k-2) \sum_{i \in V_p} \sum_{j \in V_l} x_{ji} \leq k - 1 \quad p \neq l \quad p, l = 2, \ldots, k \tag{14}$$

$$u_p \geq \sum_{j \in V_p} x_{1j} + 2 \sum_{i \in V_p} \sum_{j \in V \setminus (V_p \cup V_1)} x_{ji} \quad p = 2, \ldots, k \tag{15}$$

$$u_p \leq (k-1) \sum_{i \in V_p} x_{i1} + (k-2) \sum_{i \in V_p} \sum_{j \in V \setminus (V_p \cup V_1)} x_{ij} - (k-3) \sum_{j \in V_p} x_{1j} \quad p = 2, \ldots, k \tag{16}$$

$$u_p \geq 0 \quad p = 2, \ldots, k \tag{17}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V_p \quad \forall j \in V \setminus V_p \quad p = 1, \ldots, k \tag{18}$$

Constraints (14) prevent subtours between clusters. Constraints (15) and (16) are bounding constraints that limit the order of clusters. This limitation is realized by the k parameter in Constraints (16). Constraints (17) are non-negativity constraints. Constraints (18) are binary constraints.

3.1.1.3. Formulation SFN-N: flow-based auxiliary variable defined between nodes. We propose FB MIP formulation for the SGTSP by defining a new auxiliary variable.

In addition to the decision variables defined above, we define the following auxiliary variable:

The f_{ij} variable refers to a sequence from node i to node$j$ in case there is a transition between nodes i and j.

With the above definition, the SFN-N formulation of the SGTSP is given as:

Objective function (1)

Subject to the Constraints (2)–(8)

and

$$f_{1j} = x_{1j} \qquad j = 2, \ldots, n \tag{21}$$

$$\sum_{i \in V_p} \sum_{j \in V \backslash V_p} f_{ij} - \sum_{j \in V \backslash V_p} \sum_{i \in V_p} f_{ji} = \sum_{j \in V \backslash V_p} \sum_{i \in V_p} x_{ji} \quad p = 2, \ldots, k \tag{22}$$

$$f_{ij} \le k\, x_{ij} \quad \forall i \in V_p \quad \forall j \in V \backslash V_p \quad p = 1, \ldots, k \tag{23}$$

$$f_{ij} \ge 0 \quad \forall i \in V_p \quad \forall j \in V \backslash V_p \quad p = 1, \ldots, k \tag{24}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V_p \quad \forall j \in V \backslash V_p \quad p = 1, \ldots, k \tag{25}$$

Constraints (21) allow the starting arc to begin at node 1. Constraints (22) prevent subtours and allows the sequence number to increase cumulatively. Constraints (23) are bounding constraints that guarantee if arc (i, j) is not on any route, then the corresponding variable will be zero ($f_{ij} = 0$). The f_{ij} variable can be a maximum of k since there are $k$clusters in the problem. Constraints (21), (22), and (23) allow the sequence numbers of the selected arcs to have a value between 1 and k. Constraints (24) are non-negativity constraints. Constraints (25) are binary constraints.

3.1.1.4. Formulation SFC-C: flow-based auxiliary variable defined between clusters. We propose another FB MIP formulation for the SGTSP by defining a new auxiliary variable. Formulation SFC-C is different due to its subtour elimination and bounding constraints, which prevent subtours between clusters.

In addition to the decision variables defined above, we define the following auxiliary variable:

The f_{pl} variable refers to a sequence from cluster p to cluster$l$ in case there is a transition between clusters p and l.

With the above definition SFC-C formulation of the SGTSP is given as:

Objective function (1)

Subject to the Constraints (2)–(8)

and

$$f_{1p} = \sum_{j \in V_p} x_{1j} \qquad p = 2, \ldots, k \tag{26}$$

$$\sum_{\substack{l=1 \\ p \neq l}}^{k} f_{pl} - \sum_{\substack{l=1 \\ p \neq l}}^{k} f_{lp} = \sum_{i \in V \setminus V_p} \sum_{j \in V_p} x_{ij} \qquad p = 2, \ldots, k \tag{27}$$

$$f_{pl} \leq k \sum_{i \in V_p} \sum_{j \in V_l} x_{ij} \qquad p \neq l \qquad p = 1, \ldots, k \qquad l = 1, \ldots, k \tag{28}$$

$$f_{pl} \geq 0 \quad p \neq l \quad p = 1, \ldots, k \quad l = 1, \ldots, k \tag{29}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V_p \quad \forall j \in V \setminus V_p \quad p = 1, \ldots, k \tag{30}$$

Constraints (26) allow the starting cluster to begin at node 1. Constraints (27) prevent subtours between clusters and allows the sequence number to increase cumulatively. Constraints (28) are bounding constraints that guarantee if arc (i, j) is not on any route, then the corresponding variable will be zero (f_{pl}= 0). Thef_{pl}variable can be a maximum of k since there are k clusters in the problem. Constraints (26), (27), and (28) allow the sequence numbers of the selected arcs to have a value between 1 and k. Constraints (29) are non-negativity constraints. Constraints (30) are binary constraints.

3.1.2. Time-based formulations

3.1.2.1. Formulation TNN-N: node-based auxiliary variable defined between nodes.

We propose NB MIP formulation for the SGTSP by defining a new auxiliary variable.

The u_i variable refers to the time it takes to travel from the depot to node i.

With the above definition, TNN-N formulation of the SGTSP is given as:

Objective function (1)

Subject to the Constraints (2)–(8)

and

$$u_i - u_j + (T_{max} + t_{ij})x_{ij} + (T_{max} - t_{ji})x_{ji} \leq T_{max} \quad \forall i \in V_p \quad \forall j \in V \setminus (V_p \cup V_1) \quad p = 2, \ldots, k \tag{31}$$

$$u_i \geq \sum_{j \in V \setminus V_p} t_{ji} x_{ji} \qquad \forall i \in V_p \qquad p = 2, \ldots, k \tag{32}$$

$$u_i \leq \sum_{j \in V \setminus V_p} (T_{max} - t_{ij}) x_{ij} \qquad \forall i \in V_p \qquad p = 2, \ldots, k \tag{33}$$

$$u_i \leq T_{max} - (T_{max} - t_{1i}) x_{1i} \qquad \forall i \in V_p \qquad p = 2, \ldots, k \tag{34}$$

$$u_i \geq 0 \qquad i = 2, \ldots, n \tag{35}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in V_p \quad \forall j \in V \setminus V_p \qquad p = 1, \ldots, k \tag{36}$$

Constraints (31) prevent subtours between clusters using nodes. Constraints (32), (33), and (34) are the bounding constraints. These constraints limit the arrival time to the nodes. Constraints (35) are non-negativity constraints. Constraints (36) are binary constraints.

3.1.2.2. Formulation TNC-C: node-based auxiliary variable between clusters.

We propose another NB MIP formulation for the SGTSP by defining a new auxiliary variable. The difference between this and Formulation TNN-N is in the subtour elimination and bounding constraints, which prevent subtours between clusters.

In addition to the decision variables defined above, we define the following auxiliary variable:

The u_p variable refers to the time from the depot to cluster.

Given the above definition, TNC-C formulation of the SGTSP is given as:

Objective function (1)

Subject to the Constraints (2)–(8)

and

$$u_p - u_l + \sum_{i \in V_p} \sum_{j \in V_l} (T_{max} + t_{ij}) x_{ij} + \sum_{i \in V_p} \sum_{j \in V_l} (T_{max} - t_{ji}) x_{ji} \leq T_{max} \quad p \neq l \quad p, l = 2, \ldots, k \tag{37}$$

$$u_p \geq \sum_{i \notin V_p} \sum_{j \in V_p} t_{ij} x_{ij} \qquad p = 2, \ldots, k \tag{38}$$

$$u_p \leq \sum_{i \in V_p} \sum_{j \notin V_p} (T_{max} - t_{ij}) x_{ij} \qquad p = 2, \ldots, k \tag{39}$$

$$u_p \leq T_{max} - \sum_{i \in V_p} (T_{max} - t_{1i}) x_{1i} \qquad p = 2, \ldots, k \tag{40}$$

$$u_p \geq 0 \qquad p = 2, \ldots, k \tag{41}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V_p \quad \forall j \in V \backslash V_p \quad p = 1, \ldots, k \tag{42}$$

Constraints (37) prevent subtours between clusters. Constraints (38), (39), and (40) are the bounding constraints. These constraints limit the arrival time to the clusters. Constraints (41) are non-negativity constraints. Constraints (42) are binary constraints.

3.1.2.3. Formulation TFN-N: flow-based auxiliary variable defined between nodes.

We propose FB MIP formulation for the SGTSP by defining a new auxiliary variable.

In addition to the decision variables defined above, we define the following auxiliary variable:

f_{ij} variable refers to time from node i to node$j$ in case there is a transition between nodes i and j.

With the above definition, TFN-N formulation of the SGTSP is given as:

Objective function (1)

Subject to the Constraints (2)–(8)

and

$$f_{1j} = t_{1j}x_{1j} \qquad j = 2, \ldots, n \tag{43}$$

$$\sum_{i \in V_p}\sum_{j \in V \setminus V_p} f_{ij} - \sum_{j \in V \setminus V_p}\sum_{i \in V_p} f_{ji} = \sum_{i \in V_p}\sum_{j \in V \setminus V_p} t_{ij}x_{ij} \qquad p = 2, \ldots, k \tag{44}$$

$$f_{ij} \leq T_{max}\, x_{ij} \quad \forall i \in V_p \qquad \forall j \in V \setminus V_p \qquad p = 1, \ldots, k \tag{45}$$

$$f_{ij} \geq 0 \qquad \forall i \in V_p \qquad \forall j \in V \setminus V_p \qquad p = 1, \ldots, k \tag{46}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in V_p \qquad \forall j \in V \setminus V_p \qquad p = 1, \ldots, k \tag{47}$$

Constraints (43) allow the starting arc to begin at node 1. Constraints (44) prevent subtours and allows the time to increase cumulatively. Constraints (45) are bounding constraints that guarantee if arc (i, j) is not on any route, then the corresponding variable will be zero ($f_{ij} = 0$). The f_{ij} variable can be a maximum of T_{max} value. Constraints (46) are non-negativity constraints. Constraints (47) are binary constraints.

3.1.2.4. Formulation TFC-C: flow-based auxiliary variable defined between clusters.

We propose another FB MIP formulation for the SGTSP by defining a new auxiliary variable. The difference with 'Formulation TFN-N' is in the subtour elimination and bounding constraints that prevent subtours between clusters.

In addition to the decision variables defined above, we define the following auxiliary variable:

The f_{pl} variable refers to the arrival time from cluster p to cluster l in case there is a transition between clusters p and l.

Given the above definition, the TFC-C formulation of the SGTSP is given as:

Objective function (1)

Subject to the Constraints (2)–(8)

and

$$f_{1p} = \sum_{j \in V_p} t_{1j}x_{1j} \qquad p = 2, \ldots, k \tag{48}$$

$$\sum_{\substack{l=1 \\ p \neq l}}^{k} f_{pl} - \sum_{\substack{l=1 \\ p \neq l}}^{k} f_{lp} = \sum_{i \in V_p}\sum_{j \in V \setminus V_p} t_{ij}x_{ij} \qquad p = 2, \ldots, k \tag{49}$$

$$f_{pl} \leq T_{max} \sum_{i \in V_p}\sum_{j \in V_l} x_{ij} \quad p \neq l \qquad p = 1, \ldots, k \qquad l = 1, \ldots, k \tag{50}$$

$$f_{pl} \geq 0 \qquad p \neq l \qquad p = 1, \ldots, k \qquad l = 1, \ldots, k \tag{51}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in V_p \qquad \forall j \in V \setminus V_p \qquad p = 1, \ldots, k \tag{52}$$

Constraints (48) allow the starting cluster to begin at node 1. Constraints (49) prevent subtours between clusters and allows the time to increase cumulatively.

Constraints (50) are bounding constraints that guarantee if the arc (i, j) is not on any route, then the corresponding variable will be zero ($f_{pl} = 0$). The f_{pl} variable can be a maximum of T_{max}. Constraints (51) are non-negativity constraints. Constraints (52) are binary constraints.

All Node-Based formulations (SNN-N, SNC-C, TNN-N, TNC-C) use Miller-Tucker-Zemlin (MTZ) [27] related/based sub-tour elimination constraints. And all Flow-Based formulations (SFN-N, SFC-C, TFN-N, TFC-C) use Gavish-Graves (GG) [28] related-based sub-tour elimination constraints. All sub-tour elimination constraints are adapted for the SGTSP and all bounding constraints are proposed in this study.

The number of continuous/binary variables and constraints as a function of n (number of nodes) and k (number of clusters) for each formulation are given in Table 1.

4. Special cases of the proposed formulations

In this section, it is shown that the formulations of the Generalized Routing Problems (SOP, GTSP) being studied can be obtained by making simple changes in the objective function and the constraints of the formulations proposed for the SGTSP. Since these changes are made in the general formulation of the SGTSP given in (1)–(8), these updates are valid for all proposed formulations.

4.1. SOP

In the study conducted by Archetti et al. [21], SOP was introduced. SOP groups nodes into mutually exclusive clusters and associates with each profit that may only be collected if at least one node from the cluster is visited. SOP aims to find the route that maximizes the profits collected so that it does not exceed a certain threshold of the tour's duration. The main difference between SGTSP discussed in this study and SOP definitions is that a profit is associated with each cluster in SOP while a profit is associated with each node in SGTSP. It is sufficient to visit at least one node from the cluster in order to get a profit from the cluster in the SOP, whereas in SGTSP, only the profit of the visited node is collected.

The SOP can be solved with the mathematical model obtained by making the following changes to the general formulation presented in Section 3.1 for SGTSP. Before we present the new constraint and objective function to be added to the general formulation,

Table 1. The size of each formulation.

Formulation	# of Continuous Variables	# of Binary Variables	# of Constraints
SFC-C	k^2	n^2	$k^2 + 5k+n + 3$
SNC-C	k	n^2	$k^2 + 5k+n + 3$
SFN-N	n^2	n^2	$n^2 + 4k+2n+3$
SNN-N	n	n^2	$n^2 + 3n+3k+3$
TFC-C	k^2	n^2	$k^2 + 5k+n + 3$
TNC-C	k	n^2	$k^2 + 6k+n + 3$
TFN-N	n^2	n^2	$n^2 + 4k+2n+3$
TNN-N	n	n^2	$n^2 + 4n+3k+3$

it is necessary to define a new parameter and a decision variable:

y_p: binary variable equal to 1 if cluster p is visited, and 0 otherwise,

s_p: profit of cluster p.

In order to find the optimal solution for the SOP, the following objective function is used instead of objective function (1) given in the general formulation for SGTSP.

$$\text{Maximize} \sum_{p=2}^{k} s_p y_p \tag{53}$$

Additionally, the new constraints below must be added to the formulations:

$$y_p \leq \sum_{i \in V \setminus V_p} \sum_{j \in V_p} x_{ij}, p = 2, \ldots, k \tag{54}$$

$$y_p \in \{0, 1\}, p = 2, \ldots, k \tag{55}$$

4.2. GTSP

There is a variant of the TSP known as the GTSP in which a tour does not visit all of the nodes since set V of nodes is divided into k clusters, $V_1, \ldots, V_k$ with $V_1 \cup \ldots \cup V_k = V$ and $V_p \cap V_l = \emptyset$ if $p \neq l$. The objective is to find a minimum length tour that passes through exactly one node from each cluster, V_p.

When the SGTSP for the large T_{max} value is able to visit all clusters is given, the traveller will visit all clusters to maximize its profit. In order to solve GTSP with the formulations of the SGTSP, the T_{max} value in constraint (6), which limits the tour time, must be sufficiently large or this constraint can be omitted. By defining a new parameter, c_{ij}, the cost of travelling from node i to j, the new objective function to solve the GTSP is given below.

$$\text{Minimize} \sum_{p=1}^{k} \sum_{i \in V_p} \sum_{j \in V \setminus V_p} c_{ij} x_{ij} \tag{56}$$

Instead of constraints (4) and (5) in the general formulation for SGTSP, constraints (57) and (58) allow each cluster to be visited and can be used to solve GTSP.

$$\sum_{i \in V_p} \sum_{j \in V \setminus V_p} x_{ij} = 1, p = 2, \ldots, k \tag{57}$$

$$\sum_{i \in V \setminus V_p} \sum_{j \in V_p} x_{ij} = 1, p = 2, \ldots, k \tag{58}$$

5. Analysis of results

5.1. Discussion of formulation structure

The proposed formulations are discussed in terms of suitability for triangle inequality and special cases in the problem data set. If triangle inequality is not ensured in the time matrix of the problem, sub-optimal solutions may be obtained. The TNN-N and TNC-C models proposed above can give a sub-optimal solution in a problem data set where triangular inequality is violated.

5.1.1. Triangle inequality

Constraints (32) and (33) given above for the Formulation TNN-N model for the SGTSP can be reformulated as follows:

$$u_i \geq t_{1i}x_{1i} + \sum_{j \in V \setminus (V_p \cup V_l)} (t_{1j} + t_{ji})x_{ji} \quad \forall i \in V_p \qquad p = 2, \ldots, k \tag{59}$$

$$u_i \leq (T_{\max} - t_{i1})x_{i1} + \sum_{j \in V \setminus (V_p \cup V_l)} (T_{max} - t_{ij} - t_{j1})x_{ij} \quad \forall i \in V_p \qquad p = 2, \ldots, k \tag{60}$$

Similarly, constraints (38) and (39) given above for the Formulation TNC-C model for the SGTSP can be reformulated as follows:

$$u_p \geq \sum_{j \in V_p} t_{1j}x_{1j} + \sum_{\substack{i \notin V_p \\ i \neq 1}} \sum_{j \in V_p} (t_{1i} + t_{ij})x_{ij} \qquad p = 2, \ldots, k \tag{61}$$

$$u_p \leq \sum_{i \in V_p} (T_{\max} - t_{i1})x_{i1} + \sum_{i \in V_p} \sum_{\substack{j \notin V_p \\ j \neq 1}} (T_{max} - t_{ij} - t_{j1})x_{ij} \qquad p = 2, \ldots, k \tag{62}$$

Constraints (59), (60), (61), and (62) are tighter and thus improve the linear relaxation values of the model. It is obvious that models with better linear relaxation values can be solved in a shorter time [29]. However, these constraints do not guarantee an optimal solution when the problem data does not conform to the rule of triangular inequality. Therefore, in the case that these constraints are used, whether the rule of triangle inequality has been violated should be considered. The problem of the triangular inequality violation rule is explained in the following example (Figure 3):

In any feasible solution, either the first or second term will have the value in constraints (59). In other words, any node i will be passed directly either from starting node 1 ($x_{1i} = 1$) or from another node j ($x_{ji} = 1$). In case there is a transition from starting node 1 to node i, the $u_i \geq t_{1i}$ term in constraints (59) become valid. If there is a transition from any node j,

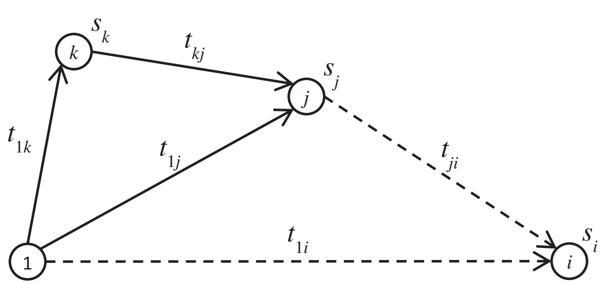

Figure 3. Triangle inequality example.

the $u_i \geq t_{1j} + t_{ji}$term becomes valid. If $x_{ji} = 1$, then $t_{1k} + t_{kj} < t_{1j}$ if the triangle inequality generated by *1kj* nodes is not satisfied. Thus, the constraint $u_i \geq t_{1j} + t_{ji} > t_{1k} + t_{kj} + t_{ji}$ becomes valid, and this constraint prevents visiting node i through the more profitable node k in a shorter time, leading to a sub-optimal solution. In the case of $x_{ji} = 1$, the satisfaction of triangle inequality (i.e., $t_{1j} + t_{ji} < t_{1i}$), which is generated by the *1ji* nodes, is not essential. Similar explanations can be applied to constraints (61) of Formulation TNC-C. Considering constraints (60) of Formulation TNN-N and constraints (62) of Formulation TNC-C, the transition from node i to either starting node $1(x_{i1} = 1)$ or another node j $(x_{ij} = 1)$ will occur. Sub-optimal solutions do not arise when the procedures described above regarding triangle inequality are applied to these constraints. As a result, when this special case is considered, it is important to satisfy the rule of triangle inequality only between the nodes and the depot. This special case is not encountered in all of the SB formulations and TB formulations of the SGTSP with an FB auxiliary variable because triangle inequality depends on the time matrix of the problem. The reason that the SB formulation approach is unaffected by this situation is that the nodes are sorted on the tour without using time values, unlike for TB formulations.

5.1.2. Special cases in problem data

In the case that nodes have the same coordinates, the subtour elimination constraints (constraints (31) and (37)) of the TB formulations (TNN-N and TNC-C) are unable to prevent subtours. For example, constraints (31) are written for both (i,j) and (j,i) such that$u_i - u_j + (T_{max} + t_{ij})x_{ij} + (T_{max} - t_{ji})x_{ji} \leq T_{max}$ and $u_j - u_i + (T_{max} + t_{ji})x_{ji} + (T_{max} - t_{ij})x_{ij} \leq T_{max}$. If the coordinates of nodes i and j are the same, then the time to travel between nodes is equal to zero, hence, $t_{ij} = t_{ji} = 0$. Constraints (31) then become $u_i - u_j + T_{max}x_{ij} + T_{max}x_{ji} \leq T_{max}$ and $u_j - u_i + T_{max}x_{ji} + T_{max}x_{ij} \leq T_{max}$. In this situation, either x_{ij} or x_{ji} is equal to 1. This concludes $u_i - u_j \leq 0$ and $u_j - u_i \leq 0$. These two inequalities will be valid if and only if $0 \leq u_i = u_j \leq T_{\max}$. According to constraints (4) and (5), each node must be visited once at most. Due to the objective function's maximization structure, the formulation will prefer to visit as many nodes as possible within the T_{max} value. In the case that nodes i and j (which have identical coordinates but different profits) are visited in a tour without a subtour, the total tour time may exceed the T_{max} value. However, it will be more profitable to visit those nodes within a subtour, and this does not change the total tour time. In conclusion, if the problem data has identical nodes, then the optimal solution (obtained by TNN-N and TNC-C formulations) may include subtours for each identical node. The disadvantages of these formulations can be prevented by combining the identical nodes into a node with the total profits of these nodes.

5.2. Computational analysis

In this section, we present the computational results of the tests we made in order to evaluate the performance of eight formulations. These formulations are coded in OPL and solved using the ILOG CPLEX 12.6 on an Intel Core i7-4770 3.4 GHz (8 cores) processor with 8 GB (1600 MHz) of RAM. All experimental runs are limited to three-hours (10,800 seconds). CPLEX is run in parallel mode using up to 8 threads. In the following section, we describe how instances are generated for the SGTSP and the computational results are provided in Section 5.2.2.

5.2.1. Test problems

We used two sets of problem instances for the computational experiments. The instances in these sets were generated through an adaptation of the existing instances for the Generalized Vehicle Routing Problem (GVRP) library [26]. Each set has 72 problems. The first set is composed of medium-sized instances and derived using the A, B and P instances in the Capacitated Vehicle Routing Problem (CVRP) library with the number of vertices being anywhere from 16 to 101. The second set consists of larger-sized instances derived from the M and G instances in the CVRP-library, encompassing 101 to 262 vertices [26].

The test instances used in the SGTSP experiments have the same number of nodes, clusters, node coordinates and cluster sets as in the GVRP test instances. The node demands in the GVRP instances correspond to the node profits in the SGTSP instances. Some of the problem data has nodes with identical coordinates. As indicated in Section 5.1.2, subtour elimination constraints in TNN-N and TNC-C formulations are unable to prevent subtours. To overcome this problem, nodes with the same coordinates are differentiated randomly. Each test instance is solved with four different T_{max} values, 100, 200, 300, and 400. In the total we have 576 (4*2*72) test instances which are available at http://www.baskent.edu.tr/~bkececi/userfiles/file/SGTSP/Instances.zip.

5.2.2. Results

This section comparatively investigates the performances of the eight mathematical formulations. We analyse computational results using the best upper bound value, CPU time (seconds), the number of optimal solutions found and the best integer objective value averaged over all instances for each test problem. The best upper bound value is the best objective function value of the LP relaxation during branch-and-cut procedure. The best integer objective value is the highest objective function value of the integer feasible solutions. Figure 4 summarizes the computational results of 4608 (576*8) runs. The values (average or total) obtained according to the performance indicators given above as a result of all experiments are shown in Figure 4. All solution values are available at http://www.baskent.edu.tr/~bkececi/userfiles/file/SGTSP/Solutions.zip.

As it is seen in Figure 4, overall evaluation SFN-N and TFN-N seem outperforming to the other formulations according to all indicators. The models were compared using these values in detail from different perspectives. Firstly, in Table 2, SB and TB formulations were evaluated by comparing N_N and C_C. When the values in Table 2 were obtained, the averages (sum for #OptSol indicator) of the FB and NB formulations were taken. For example, 1329.4 was calculated by taking the average of the values obtained from the SFC-C and SNC-C formulations. The values in Table 2 show that all formulations are close to each other according to the Avg. BestInt and Avg. BestUB criteria. When the SB formulations were evaluated, it was found that the Avg. CPU Time of the C_C formulations (SNC-C and SFC-C) was smaller than the average of N_N formulations (SNN-N and SFN-N). However, when TB formulations were evaluated, there was no difference between these values. 11 more optimal solutions were found by the SB formulations using the C_C auxiliary variable definition (SNC-C and SFC-C

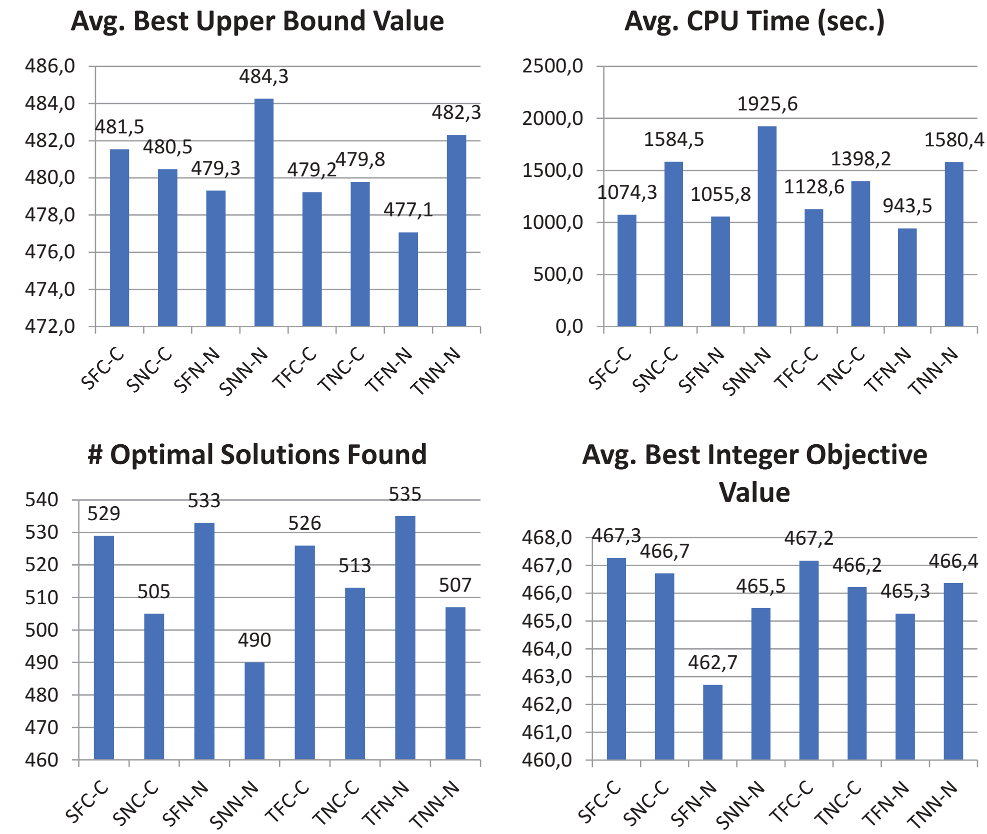

Figure 4. Results of performance indicators.

formulations) than were obtained using the N_N auxiliary variable definition (SNN-N and SFN-N formulations). However, this result has been otherwise achieved for TB formulations. Three more optimal solutions were found by the TB formulations using the N_N auxiliary variable definition (TNN-N and TFN-N formulations) than were obtained using the C_C auxiliary variable definition (TNC-C and TFC-C formulations).

In Table 3, SB and TB formulations are evaluated in comparison with the definition of FB and NB auxiliary variables. When the values in Table 3 were obtained, the averages of C_C and N_N formulations (sums for #OptSol indicator) were calculated. For example, a value of 1065.0 was calculated by taking the average of the values obtained from the SFC-C and SFN-N formulations. The values in Table 3 show that all formulations were close to each other according to the Avg. BestInt and Avg. BestUB criteria. The number

Table 2. Formulation comparison of SB vs. TB and N_N vs. C_C.

	Avg. CPU Time (sec.)		# Opt. Solution Found		Avg. Best Integer Objective Value		Avg. Best Upper Bound Value	
	C_C	N_N	C_C	N_N	C_C	N_N	C_C	N_N
SB	1,329.4	1,490.7	1,034	1,023	467.0	464.1	481.0	481.8
TB	1,263.4	1,262.0	1,039	1,042	466.7	465.8	479.5	479.7

Table 3. Formulation comparison of SB vs. TB and NB vs. FB.

	Avg. CPU Time (sec.)		# Opt. Solution Found		Avg. Best Integer Objective Value		Avg. Best Upper Bound Value	
	FB	NB	FB	NB	FB	NB	FB	NB
SB	1,065.0	1,755.1	1,062	995	465	466.1	480.3	482.4
TB	1,036.1	1,489.3	1,061	1,020	466.2	466.3	478.2	481.0

of optimal solutions found through formulations (SFC-C and SFN-N, TFN-N and TFC-C) defined by FB auxiliary variable was higher than the formulations defined by the NB auxiliary variable. Similarly, according to the Avg. CPU Time indicator, it may be seen that the formulations defined by the FB auxiliary variable obtained better results than the formulations defined by the NB auxiliary variable.

In Table 4, the C_C and N_N formulations are compared with the definitions of FB and NB auxiliary variables. When the values in Table 4 were obtained, the averages of SB and TB formulations (sums for #OptSol) were calculated. For example, a value of 1101.4 was obtained by taking the average of the values obtained from the SFC-C and TFC-C formulations. The values in Table 4 show that all formulations are close to each other according to the Avg. BestInt and Avg. BestUB criteria. The definition of FB auxiliary variables in both of C_C and N_N formulations provided a more optimal solution in a shorter time than NB did. 37 more optimal solutions were found by the C_C formulations using the definition of FB auxiliary variables (SFC-C and TFC-C formulations) than were obtained by the definition of NB auxiliary variables (SNC-C and TNC-C formulations). Similarly, 71 more optimal solutions were found by the N_N formulations obtained using the definition of FB auxiliary variables (SFN-N and TFN-N formulations) than were obtained by the definition of NB auxiliary variables (SNN-N and TNN-N formulations).

Finally, in Table 5, SB and TB formulations, C_C and N_N formulations, and NB and FB formulations were evaluated separately in comparison to the determined performance criteria. In rows 1 and 2, the averages (sums for #OptSol) of four SB formulations (SNN-N, SFN-N, SNC-C and SFC-C) and the averages of four TB formulations (TNN-N, TFN-N, TNC-C and TFC-C) were calculated. In rows 3 and 4, the averages (sums for #OptSol) of four C_C formulations (SNC-C, SFC-C, TNC-C and TFC-C) and N_N formulations (SNN-N, SFN-N, TNN-N and TFN-N) were calculated. Finally, in the last two rows, the averages (sums for #OptSol) of four FB formulations (SFN-N, SFC-C, TFN-N and TFC-C) and NB formulations (SNN-N, SNC-C, TNN-N and TNC-C) were calculated. The values in Table 4 show that all formulations are close to each other according to the Avg. BestInt and Avg. BestUB criteria. In Table 5, in terms of the criteria Avg. CPU Time and the number of optimal solutions found, TB formulations are superior to SB, C_C formulations are superior to N_N, and FB formulations are superior to NB.

Table 4. Formulation comparison of C_C vs. N_N and FB vs. NB.

	Avg. CPU Time (sec.)		# Opt. Solution Found		Avg. Best Integer Objective Value		Avg. Best Upper Bound Value	
	FB	NB	FB	NB	FB	NB	FB	NB
C_C	1,101.4	1,491.4	1,055	1,018	467.2	466.5	480.4	480.1
N_N	999.7	1,753.0	1,068	997	464.0	465.9	478.2	483.3

Table 5. Formulation comparison of SB vs. TB, C_C vs. N_N, and FB vs. NB.

	Avg. CPU Time (sec.)	# Opt. Solution Found	Avg. Best Integer Objective Value	Avg. Best Upper Bound Value
SB	1,410.1	2,057	465.5	481.4
TB	1,262.7	2,081	466.3	479.6
C_C	1,296.4	2,073	466.8	480.3
N_N	1,376.4	2,065	465.0	480.7
FB	1,050.5	2,123	465.6	479.3
NB	1,622.2	2,015	466.2	481.7

Figure 5 shows the change in the average CPU time according to T_{max} for all formulations. It may obviously be seen in Figure 5 that all formulations need more CPU time to solve the problem of T_{max} = 200. SB formulations need the least CPU time when T_{max} = 400; however, TB formulations need the least CPU time when T_{max} = 100. Figure 6 shows the change in the total number of optimal solutions found by the formulations with different T_{max} values and supports the above results. In SB formulations, the maximum total number of optimal solutions that may be found is reached when T_{max} = 400. However, in TB formulations, the maximum total number of optimal solutions found is reached when T_{max} = 100.

Figure 7 gives the dominance relationships between all of the LP relaxations of different formulations. In this figure the average percentage gap values are used. Gap value is calculated by using (U − L)/L, where U is the value of LP relaxation and L is the value of either the best or the optimal solution obtained within the time limit. And the percentage gap values are averaged over all instances and different T_{max} values. The smaller the average percentage gap value, the better the LP relaxation is.

It is observed in Figure 7 that we can divide formulations in three categories from the best to worst according to average percentage gap values. The Time-based formulations with Flow-based auxiliary variable give best LP relaxation values. And the formulations with Node-based auxiliary variables which are defined between nodes have the worst LP relaxation values.

All results obtained with eight formulations are given in Appendix A. The tables in the appendix include all results according to different T_{max} values and two sets of instances (Set1, Set2). The asterisk in the tables is used to mark the solutions which exceed time limit (10800 sec.). In overall evaluation, time-based formulations can find more optimal solutions in less computation time. Besides, sequence based formulations have the advantage of being not affected by triangular inequality and the identical coordinates of nodes.

In addition to the above analysis, we compare our eight formulations with the formulation proposed in [21] and [30]. To be able to solve the problems in [21] we changed the objective function (1) with the function given in (63), and we add the constraint (64) to the general formulation. In (63) the s_p is the profit of cluster p and the y_p is a binary variable where it takes 1 if the cluster p is visited. The proposed formulation (ILP) in [30] leads to noticeably faster solutions than [21]. That is why in our comparison tables we only use the results of ILP in [30]. And these results are given in Table 6. In this table the first column shows the instance name, and the consecutive two columns show the BestInt and %Gap values for each formulation, and the last column gives the %Gap value of the ILP formulation proposed in [30].

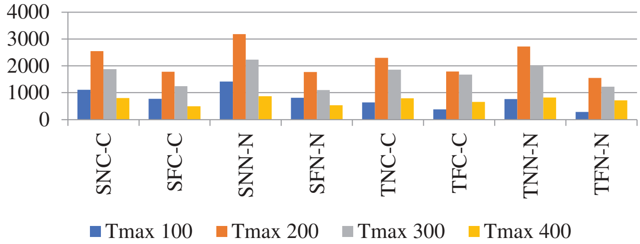

Figure 5. CPU Time results of different T_{max} values.

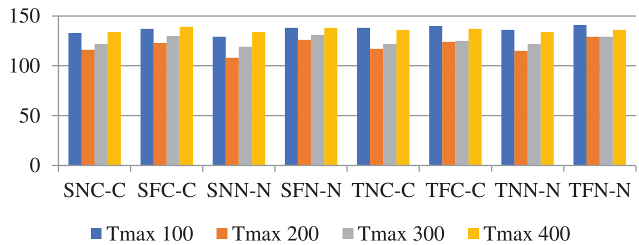

Figure 6. Number of optimal solutions found with different T_{max} values.

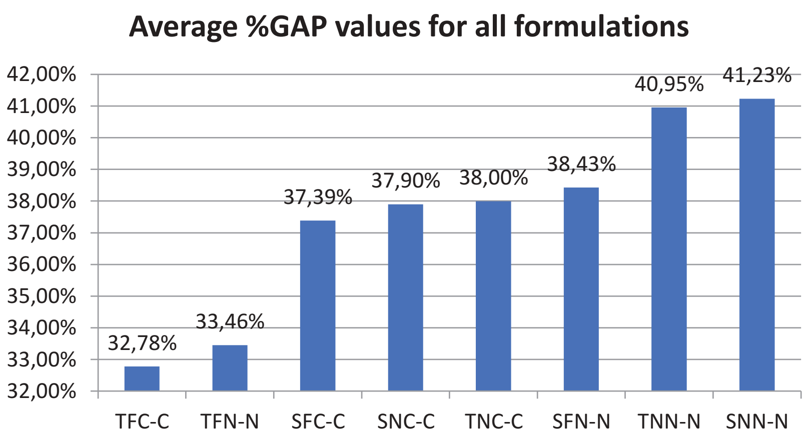

Figure 7. Average %GAP values for all formulations.

$$\text{Maximize} \sum_{p=2}^{k} s_p y_p \tag{63}$$

$$y_p \leq \sum_{i \in V \setminus V_p} \sum_{j \in V_p} x_{ij} \quad p = 2, \ \ldots, k \tag{64}$$

According to the results given in Table 6, ILP can optimally solve 24 out of 88 instances; however our SFC-C formulation can optimally solve 29 out of 88 instances. Moreover, we considered 10 more problem instances in this comparison and we find the optimal

Table 6. Comparison of all formulations with ILP.

	SNN-N		SNC-C		SFN-N		TNN-N		TNC-C		TFN-N		TFC-C		SFC-C		ILP
Instance	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	Gap(%)
11berlin52_T100_p1	50	0	50	0	50	0	50	0	50	0	50	0	50	0	50	0	-
11berlin52_T100_p2	2580	0	2580	0	2580	0	2580	0	2580	0	2580	0	2580	0	2580	0	-
11berlin52_T40_p1	37	0	37	0	37	0	37	0	37	0	37	0	37	0	37	0	0
11berlin52_T40_p2	1829	0	1829	0	1829	0	1829	0	1829	0	1829	0	1829	0	1829	0	0
11berlin52_T60_p1	43	0	43	0	43	0	43	0	43	0	43	0	43	0	43	0	0
11berlin52_T60_p2	2190	0	2190	0	2190	0	2190	0	2190	0	2190	0	2190	0	2190	0	0
11berlin52_T80_p1	47	0	47	0	47	0	47	0	47	0	47	0	47	0	47	0	0
11berlin52_T80_p2	2384	0	2384	0	2384	0	2384	0	2384	0	2384	0	2384	0	2384	0	0
11eil51_T100_p1	48	0	48	0	48	0	48	0	48	0	48	0	48	0	48	0	-
11eil51_T100_p2	2426	0	2426	0	2426	0	2426	0	2426	0	2426	0	2421	0	2426	0	-
11eil51_T40_p1	24	0	24	0	24	0	24	0	24	0	24	0	24	0	24	0	0
11eil51_T40_p2	1279	0	1279	0	1279	0	1279	0	1279	0	1279	0	1279	0	1279	0	0
11eil51_T60_p1	39	0	39	0	39	0	39	0	39	0	39	0	39	0	39	0	0
11eil51_T60_p2	1911	0	1911	0	1911	0	1911	0	1911	0	1911	0	1911	0	1911	0	0
11eil51_T80_p1	43	0	43	0	43	0	43	0	43	0	43	0	43	0	43	0	0
11eil51_T80_p2	2114	0	2114	0	2114	0	2114	0	2114	0	2114	0	2114	0	2114	0	0
14st70_T100_p1	68	1.47	68	1.47	68	0	68	1.47	68	1.47	68	1.47	68	1.47	68	0	-
14st70_T100_p2	3488	0.72	3488	0.72	3488	0	3488	0.72	3488	0	3488	0.72	3488	0.72	3488	0	-
14st70_T40_p1	33	0	33	0	33	0	33	0	33	0	33	0	33	0	33	0	0
14st70_T40_p2	1672	0	1672	0	1672	0	1672	0	1672	0	1672	0	1672	0	1672	0	0
14st70_T60_p1	50	18.19	50	0	50	0	50	12.08	50	0	50	18.51	50	0	50	0	-
14st70_T60_p2	2589	16.55	2589	0	2589	0	2589	11.12	2589	0	2589	11.12	2589	7.72	2589	0	-
14st70_T80_p1	65	4.62	65	0	65	0	65	4.62	65	0	65	0	65	0	65	0	0
14st70_T80_p2	3355	3.96	3355	0	3355	0	3229	8.06	3355	0	3355	0	3355	0	3355	0	0
16eil76_T40_p1	37	0	37	0	37	0	37	0	37	0	37	0	37	0	37	0	0
16eil76_T40_p2	2094	0	2094	0	2094	0	2094	0	2094	0	2094	0	2094	0	2094	0	0
16eil76_T60_p1	56	9.98	56	0	56	0	56	9.77	56	0	56	0	56	0	56	0	0
16eil76_T60_p2	2967	5.02	2967	0	2967	0	2967	5.67	2967	0	2967	9.78	2967	0	2967	0	0

(*Continued*)

Table 6. (Continued).

	SNN-N		SNC-C		SFN-N		TNN-N		TNC-C		TFN-N		TFC-C		SFC-C		ILP
Instance	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	Gap(%)
16eil76_T80_p1	67	8.37	67	3.42	67	0	67	7.46	67	0	67	7.37	67	0	67	0	-
16eil76_T80_p2	3525	4.91	3525	1.36	3525	0	3525	4.91	3525	1.36	3525	4.91	3525	1.36	3525	0	-
16pr76_T100_p1	74	1.35	74	1.35	74	0	74	1.35	74	1.35	74	1.35	74	1.35	74	1.35	1.4
16pr76_T100_p2	3765	0.93	3765	0.93	3765	0	3765	0.93	3765	0.93	3765	0.93	3765	0.93	3765	0	0
20kroA100_T100_p1	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	0	3.1
20kroA100_T100_p2	4868	2.88	4868	2.88	4868	2.88	4868	2.88	4868	2.88	4868	2.88	4868	2.88	4868	2.88	2.9
20kroB100_T100_p1	98	1.02	98	1.02	98	1.02	98	1.02	98	1.02	98	1.02	98	1.02	98	1.02	1
20kroB100_T100_p2	4916	1.87	4916	1.87	4916	1.87	4916	1.87	4916	1.87	4916	1.87	4916	1.87	4916	1.87	1.9
20kroC100_T100_p1	95	4.21	95	4.21	97	2.06	95	4.21	95	4.21	95	4.21	97	2.06	97	0	2.1
20kroC100_T100_p2	4818	3.94	4882	2.58	4882	2.58	4818	3.94	4882	2.58	4818	3.94	4882	2.58	4882	0	2.6
20kroD100_T100_p1	94	5.32	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	3.1
20kroD100_T100_p2	4792	4.51	4838	3.51	4838	3.51	4838	3.51	4838	3.51	4774	4.9	4838	3.51	4838	2.86	3.5
20kroE100_T100_p1	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	3.12	96	0	0
20kroE100_T100_p2	4887	2.48	4887	2.48	4887	2.48	4887	2.48	4887	2.48	4869	2.85	4887	2.48	4887	2.26	0
20rat99_T100_p1	90	8.89	92	6.52	90	8.89	90	8.89	90	8.89	92	6.52	90	8.89	92	6.52	15.3
20rat99_T100_p2	4584	9.23	4670	7.22	4721	6.06	4721	6.06	4584	9.23	4627	8.21	4721	6.06	4584	9.23	11.7
20rd100_T100_p1	97	2.06	97	2.06	97	2.06	97	2.06	97	2.06	97	2.06	97	2.06	97	2.06	2.1
20rd100_T100_p2	4906	2.08	4929	1.6	4929	1.6	4929	1.6	4929	1.6	4919	1.81	4929	1.6	4929	1.6	1.6
21eil101_T100_p1	97	3.09	97	2.77	97	2.78	97	3.09	97	3.01	97	3.09	97	3.09	97	1.03	1
21eil101_T100_p2	4938	2.27	4938	2.2	4938	2.27	4938	2.27	4938	2.24	4938	2.27	4938	2.27	4938	1.56	2.1
21lin105_T100_p1	99	5.05	102	1.96	102	1.96	100	4	102	1.96	100	4	102	1.96	102	0	2
21lin105_T100_p2	5101	2.49	5157	1.38	5157	1.38	5157	1.38	5157	1.38	5157	1.38	5157	1.38	5157	0	2.5
22pr107_T100_p1	101	4.95	100	6	101	4.37	101	4.95	101	4.95	101	4.95	101	4.95	101	4.95	5
22pr107_T100_p2	5101	5.14	5101	5.14	5104	1.21	5006	7.13	5101	5.14	5101	5.14	5101	5.14	5104	5.07	5.1
25pr124_T100_p1	115	6.96	121	1.65	121	1.65	117	5.13	121	1.65	111	10.81	121	1.65	119	3.36	7.9
25pr124_T100_p2	6159	1.19	6050	3.01	6050	3.01	6050	3.01	6159	1.19	5864	6.28	6159	1.19	6159	1.19	1.2
26bier127_T100_p1	125	0.8	125	0	125	0	125	0	125	0	125	0	125	0	125	0	0.8
26bier127_T100_p2	6314	0.17	6314	0	6314	0	6314	0.18	6314	0	6314	0.17	6314	0	6314	0	0

(*Continued*)

Table 6. (Continued).

	SNN-N		SNC-C		SFN-N		TNN-N		TNC-C		TFN-N		TFC-C		SFC-C		ILP
Instance	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	Gap(%)
26ch130_T100_p1	127	1.57	127	1.57	127	1.57	127	1.57	127	1.57	126	2.38	127	1.57	127	1.57	2.4
26ch130_T100_p2	6412	1.42	6412	1.42	6365	2.17	6412	1.42	6412	1.42	6373	2.04	6288	3.42	6412	1.42	1.4
28pr136_T100_p1	134	0.75	133	1.5	133	1.5	133	1.5	133	1.5	133	1.5	133	1.5	134	0.75	0.7
28pr136_T100_p2	6808	0.62	6841	0.13	6808	0.62	6808	0.62	6808	0.62	6808	0.62	6808	0.62	6808	0.62	0.6
29pr144_T100_p1	141	1.42	141	1.42	141	1.42	141	1.42	141	1.42	139	2.88	141	1.42	141	1.42	2.9
29pr144_T100_p2	7137	1.47	7137	1.47	7137	1.47	7137	1.47	7137	1.47	7060	2.58	7137	1.47	7195	0.65	1.5
30ch150_T100_p1	144	3.47	144	3.47	144	3.47	144	3.47	144	3.47	142	4.93	144	3.47	146	2.05	11.2
30ch150_T100_p2	7130	5.65	7315	2.98	7168	5.09	7123	5.76	7315	2.98	7282	3.45	7288	3.36	7394	1.88	11.6
30kroA150_T100_p1	145	2.76	145	2.76	144	3.47	142	4.93	144	3.47	136	9.56	144	3.47	145	2.76	6.4
30kroA150_T100_p2	7361	2.34	7317	2.95	7099	6.11	7317	2.95	7361	2.34	7317	2.95	7317	2.95	7317	2.95	5.4
30kroB150_T100_p1	148	0.68	148	0.68	148	0.68	148	0.68	148	0.68	148	0.68	148	0.68	148	0.68	0.7
30kroB150_T100_p2	7445	1.18	7445	1.18	7445	1.18	7445	1.18	7445	1.18	7445	1.18	7445	1.18	7445	1.18	2.4
31pr152_T100_p1	142	6.34	142	6.34	142	6.34	115	31.3	142	6.34	131	15.27	142	6.34	142	6.34	10.2
31pr152_T100_p2	7203	6.32	7245	5.7	7203	6.32	5800	32.03	5800	32.03	6962	10	7203	6.32	7203	6.32	17
32u159_T100_p1	153	3.27	156	1.28	153	3.27	152	3.95	156	1.28	152	3.95	156	1.28	155	1.94	10.5
32u159_T100_p2	7679	4.66	7924	1.43	7913	1.57	7923	1.44	7887	1.9	7792	3.14	7887	1.9	7978	0.74	4.8
39rat195_T100_p1	186	4.3	189	2.65	181	7.18	184	5.43	184	5.43	175	10.86	185	4.86	182	6.59	18.3
39rat195_T100_p2	9414	4.77	9448	4.39	9340	5.6	9340	5.6	9558	3.19	8258	19.44	9340	5.6	9399	4.94	16.9
40d198_T100_p1	171	15.2	171	15.2	185	6.49	171	15.2	171	15.2	181	8.84	171	15.2	195	1.03	15.2
40d198_T100_p2	8628	15.87	8628	15.87	8628	15.87	8628	15.87	9924	0.74	9138	9.4	8628	15.87	8628	15.87	15.9
40kroa200_T100_p1	195	2.05	198	0.51	197	1.02	198	0.51	198	0.51	183	8.74	194	2.58	198	0.51	5.3
40kroa200_T100_p2	9832	2.3	9970	0.88	9922	1.37	9822	2.4	9970	0.88	7477	34.52	9970	0.88	9970	0.88	5
40krob200_T100_p1	193	3.11	198	0.51	196	1.53	196	1.53	198	0.51	176	13.07	196	1.53	195	2.05	3.6
40krob200_T100_p2	9898	1.62	9912	1.47	9864	1.97	9980	0.78	9990	0.68	8019	25.43	9820	2.42	9966	0.92	1.9
45ts225_T100_p1	215	4.19	221	1.36	204	9.8	210	6.67	218	2.75	149	50.34	208	7.69	218	2.75	21.1
45ts225_T100_p2	10,663	6.05	11,187	1.08	10,872	4.01	11,017	2.64	11,187	1.08	8769	28.95	11,016	2.65	11,150	1.42	15.8
45tsp225_T100_p1	187	19.79	217	3.23	178	25.84	208	7.69	213	5.16	143	56.64	212	5.66	210	6.67	20.4
45tsp225_T100_p2	10,771	4.99	10,960	3.18	9468	19.43	9607	17.71	11,002	2.78	8763	29.04	10,688	5.8	10,960	3.18	17.6

(Continued)

Table 6. (Continued).

	SNN-N		SNC-C		SFN-N		TNN-N		TNC-C		TFN-N		TFC-C		SFC-C		ILP
Instance	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	BestInt	%Gap	Gap(%)
46pr226_T100_p1	223	0.9	224	0.45	223	0.9	223	0.9	224	0.45	215	4.65	223	0.9	223	0.9	1.4
46pr226_T100_p2	11,354	0.18	11,368	0.06	11,354	0.18	11,227	1.32	11,368	0.06	11,081	2.65	11,350	0.22	11,358	0.15	1.4
53gil262_T100_p1	184	41.85	255	2.35	173	50.87	172	51.74	255	2.35	163	60.12	245	6.53	251	3.98	21.4
53gil262_T100_p2	7560	74.51	12,918	2.13	9060	45.62	7782	69.53	12,933	2.01	7411	78.02	11,299	16.76	10,851	21.58	20.4
53pr264_T100_p1	145	81.38	234	12.39	190	38.42	140	87.86	237	10.97	147	78.91	255	3.14	216	21.76	14.3
53pr264_T100_p2	13,277	0.19	13,277	0.19	8644	53.89	4249	213.06	13,187	0.87	7519	76.91	13,083	1.67	11,979	11.04	0.2
56a280_T100_p1	211	32.23	269	3.72	125	123.2	52	436.54	264	5.68	113	146.9	238	17.23	221	26.24	31.6
56a280_T100_p2	9024	57.11	13,841	2.43	7541	88.01	612	2200	13,495	5.06	6042	134.66	8846	60.28	13,435	5.53	18.2
60pr299_T100_p1	196	52.04	292	2.05	81	267.9	76	292.11	295	1.02	67	344.78	200	49	284	4.93	10.4
60pr299_T100_p2	7919	90.77	14,973	0.89	4636	225.86	163	9200	14,790	2.14	4469	238.04	10,040	50.47	12,508	20.78	24.5
64lin318_T100_p1	255	24.31	314	0.96	209	51.67	171	85.38	310	2.26	249	27.31	306	3.59	303	4.62	7.5
64lin318_T100_p2	13,754	16.6	15,860	1.12	10,679	50.17	8897	80.25	15,696	2.17	11,569	38.62	14,199	12.94	15,342	4.53	5.7
80rd400_T100_p1	42	850	390	2.31	178	124.16	173	130.64	395	1.01	6	6500	212	88.21	324	23.15	16.7
80rd400_T100_p2	7519	168.09	19,383	4	9347	115.66	10,681	88.73	19,655	2.56	366	5400	12,152	65.88	10,391	93.99	14.4
Avg.		**18.18**		**2.04**		**14.75**		**135.34**		**2.22**		**139.16**		**5.80**		**3.83**	**5.85**

solutions these instances. The average percentage gap value obtained by ILP is 5.85%, while the SFC-C, SNC-C, TNC-C and TFC-C give better percentage gap values than ILP. This shows that the auxiliary variables defined in a formulations between clusters are more efficient to solve these instances.

6. Conclusions

In this paper, we have studied the selective generalized travelling salesman problem, which is a variant of the generalized travelling salesman problem. In the selective generalized travelling salesman problem, a profit is associated with each node, and nodes are grouped in clusters. This profit is collected if a node is visited. The objective is to find the tour that maximizes the collected profit such that the corresponding duration does not exceed a given threshold. We proposed eight mathematical formulations using different types of auxiliary variables for the selective generalized travelling salesman problem. To compare and analyse these formulations, 4608 computational runs were conducted. Overall, 4138 out of 4608 (~90%) test instances were solved optimally by using all formulations, and some important insights into the definition of auxiliary variables in the formulations were revealed. In addition, our formulations are compared with the previously proposed formulations in the literature. The selective generalized travelling salesman problem may find application in the distribution of mass products. As such, this method may provide a way to organize distribution processes that provide advantages both to carriers and customers. In future studies, all variants of generalized routing problems the selective generalized travelling salesman problem should be studied. Furthermore, some heuristic and meta-heuristic approaches should be developed to obtain good quality solutions for larger problems.

Disclosure statement

No potential conflict of interest was reported by the authors.

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References

[1] D. Feillet, P. Dejax, and M. Gendreau, *Traveling salesman problems with profits*, Transp. Sci. 39 (2) (2005), pp. 188–205. doi:10.1287/trsc.1030.0079.
[2] C. Verbeeck, E.H. Aghezzaf, and P. Vansteenwegen, *Solving the stochastic time-dependent orienteering problem with time windows*, Eur. J. Oper. Res. 255 (2016), pp. 699–718. doi:10.1016/j.ejor.2016.05.031.
[3] T. Tsiligirides, *Heuristic methods applied to orienteering*, J. Oper. Res. Soc. 35 (9) (1984), pp. 797–809. doi:10.1057/jors.1984.162.
[4] P. Vansteenwegen, W. Souffriau, and D. Van Oudheusden, *The orienteering problem: A survey*, Eur. J. Oper. Res. 209 (1) (2011), pp. 1–10. doi:10.1016/j.ejor.2010.03.045.
[5] A. Gunawan, H.C. Lau, and P. Vansteenwegen, *Orienteering Problem: A survey of recent variants, solution approaches and applications*, Eur. J. Oper. Res. 255 (2016), pp. 315–332. doi:10.1016/j.ejor.2016.04.059.

[6] Z.W. Geem, C.L. Tseng, and Y. Park, *Harmony search for generalized orienteering problem: Best touring in China*, in *Lecture Notes in Computer Science*, Vol. 3612, L. Wang, K. Chen, and Y. S. Ong, eds., Advances in natural computation, Springer, Berlin, Heidelberg, 2005, pp. 741–750.
[7] X. Wang, B.L. Golden, and E.A. Wasil, *Using a genetic algorithm to solve the generalized orienteering problem*, in *Operations Research/Computer Science Interfaces*, Vol. 43, B. L. Golden, S. Raghavan, and E.A. Wasil, eds., The vehicle routing problem: Latest advances and new challenges, Springer, US, 2008, pp. 263–274.
[8] Q. Wang, X. Sun, and B.L. Golden, *Using artificial neural networks to solve generalized orienteering problems*, Intell. Eng. Syst. Artif. Neural Networks 6 (1996), pp. 1063–1068.
[9] J. Silberholz and B. Golden, *The effective application of a new approach to the generalized orienteering problem*, J. Heuristics 16 (3) (2010), pp. 393–415. doi:10.1007/s10732-009-9104-8.
[10] J. Pietz and J.O. Royset, *Generalized orienteering problem with resource dependent rewards*, Nav. Res. Logist. 60 (4) (2013), pp. 294–312. doi:10.1002/nav.v60.4.
[11] Z. Ardalan, S. Karimi, O. Poursabzi, and B. Naderi, *A novel imperialist competitive algorithm for generalized traveling salesman problems*, Appl. Soft. Comput. 26 (2015), pp. 546–555. doi:10.1016/j.asoc.2014.08.033.
[12] D. Karapetyan and G. Gutin, *Lin–Kernighan heuristic adaptations for the generalized traveling salesman problem*, Eur. J. Oper. Res. 208 (3) (2011), pp. 221–232. doi:10.1016/j.ejor.2010.08.011.
[13] D. Karapetyan and G. Gutin, *Efficient local search algorithms for known and new neighborhoods for the generalized traveling salesman problem*, Eur. J. Oper. Res. 219 (2) (2012), pp. 234–251. doi:10.1016/j.ejor.2012.01.011.
[14] K. Sundar and S. Rathinam, *Generalized multiple depot traveling salesmen problem—Polyhedral study and exact algorithm*, Comput. Oper. Res. 70 (2016), pp. 39–55. doi:10.1016/j.cor.2015.12.014.
[15] P.C. Pop, O. Matei, and C. Sabo, *A hybrid diploid genetic based algorithm for solving the generalized traveling salesman problem. in Proc. of HAIS 2017*, Lect. Notes Comput. Sci. 10334 (2017), pp. 149–160.
[16] P.C. Pop, *New integer programming formulations of the generalized traveling salesman problem*, Am. J. Appl. Sci. 4 (11) (2007), pp. 932–937. doi:10.3844/ajassp.2007.932.937.
[17] P.C. Pop and S. Iordache, *A Hybrid Heuristic Approach for Solving the Generalized Traveling Salesman Problem*. In Proceedings of the 13th annual conference on Genetic and evolutionary computation, ACM, (2011), pp. 481–488.
[18] P.C. Pop, *New Models of the Generalized Minimum Spanning Tree Problem*, J. Math. Modell. Algorithms 3 (2) (2004), pp. 153–166. doi:10.1023/B:JMMA.0000036579.83218.8d.
[19] P.C. Pop, *A survey of different integer programming formulations of the generalized minimum spanning tree problem*, Carpathian J. Math. 25 (2009), pp. 104–118.
[20] P.C. Pop, I. Kara, and A. Horvat Marc, *New Mathematical Models of the Generalized Vehicle Routing Problem and Extensions*, Appl. Math. Modell.Elsevier 36 (1) (2012), pp. 97–107. doi:10.1016/j.apm.2011.05.037.
[21] C. Archetti, F. Carrabs, and R. Cerulli, *The Set Orienteering Problem*, Eur. J. Oper. Res. 267 (2018), pp. 264–272. doi:10.1016/j.ejor.2017.11.009.
[22] P. Vansteenwegen and D. Van Oudheusden, *The mobile tourist guide: An or opportunity*, OR Insights 20 (3) (2007), pp. 21–27. doi:10.1057/ori.2007.17.
[23] G. Laporte, A. Asef-Vaziri, and C. Sriskandarajah, *Some applications of the generalized travelling salesman problem*, J. Oper. Res. Soc. 47 (12) (1996), pp. 1461–1467. doi:10.1057/jors.1996.190.
[24] L. Mou, *An efficient ant colony system for solving the new generalized traveling salesman problem*, in *2011 IEEE International Conference on Cloud Computing and Intelligence Systems (CCIS)*, IEEE, 2011, pp. 407–412. doi:10.1109/CCIS.2011.6045099.
[25] S.L. Smith and F. Imeson, *GLNS: An effective large neighborhood search heuristic for the generalized traveling salesman problem*, Comput. Oper. Res. 87 (2017), pp. 1–19. doi:10.1016/j.cor.2017.05.010.
[26] T. Bektaş, G. Erdoğan, and S. Røpke, *Formulations and branch-and-cut algorithms for the generalized vehicle routing problem*, Transp. Sci. 45 (3) (2011), pp. 299–316. doi:10.1287/trsc.1100.0352.

[27] C.E. Miller, A.W. Tucker, and R.A. Zemlin, *Integer programming formulation of traveling salesman problems*, J. Acm 7 (4) (1960), pp. 326–329. doi:10.1145/321043.321046.

[28] B. Gavish and S.C. Graves (1978). The travelling salesman problem and related problems. Working Paper GR-078-78, Operations Research Center, Massachusetts Institute of Technology.

[29] B. Kececi, T. Derya, E. Dinler, and Y.T. Ic, *A comparative study of the capability of alternative mixed integer programming formulations*, Technol. Econ. Dev. Economy 24 (2) (2017), pp. 561–584. doi:10.3846/20294913.2016.1213202.

[30] R. Penicka, F. Faigl, and M. Saska, *Variable Neighborhood Search for the Set Orienteering Problem and its application to other Orienteering Problem variants*, Eur. J. Oper. Res. 276 (2019), pp. 816–825. doi:10.1016/j.ejor.2019.01.047.

Appendix A.

Table A1. Experimental results of eight formulations with different Tmax values.

	Set1 (Tmax = 100)									Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB	BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB
A.32.17	63	63	63	63	63	63	63	63	B.45.24.1	88	88	88	88	88	88	88	88
A.33.18.1	76	76	76	76	76	76	76	76	B.50.26.1	134	134	134	134	134	134	134	134
A.33.18.2	116	116	116	116	116	116	116	116	B.50.26.2	132	132	132	132	132	132	132	132
A.34.18	48	48	48	48	48	48	48	48	B.51.27	54	54	54	54	54	54	54	54
A.36.19	70	70	70	70	70	70	70	70	B.52.27	177	177	177	177	177	177	177	177
A.37.1.20	94	94	94	94	94	94	94	94	B.56.29	216	216	216	216	216	216	216	216
A.37.20	70	70	70	70	70	70	70	70	B.57.30.1	65	65	65	65*	65	65	65	65
A.38.20	91	91	91	91	91	91	91	91	B.57.30.2	24	24	24	24	24	24	24	24
A.39.21.1	67	67	67	67	67	67	67	67	B.63.33	85	85*	85	85*	85	85	85	85
A.39.21.2	146	146	146	146	146	146	146	146	B.64.33	210	210	210	210	210	210	210	210
A.44.23	102	102	102	102	102	102	102	102	B.66.34	97	97*	97*	97*	97	97*	97	97
A.45.24.1	70	70	70	70	70	70	70	70	B.67.35	142*	143*	145	142*	145	145	145	145*
A.45.24.2	81	81	81	81	81	81	81	81	B.68.35	104	104	104	104	104	104	104	104
A.46.24	95	95	95	95	95	95	95	95	B.78.40	164*	164*	164	164*	164	164	164	164*
A.48.25	68	68	68	68	68	68	68	68	G.262.132	470	470*	470	470*	470	470	470	470
A.53.28	68	68	68	68	68	68	68	68	M.101.52	300*	300*	300	300*	300	300*	300	290*
A.54.28	99	99	99	99	99	99	99	99	M.121.62	213*	215*	216*	208*	218*	216*	218	204*
A.55.29	106	106	106	106	106	106	106	106	M.151.77	415	415	415	415	415	415	415	415
A.60.31	95	95	95	95	95	95	95	95	M.200.101	461*	481*	439*	483*	483*	483*	478*	482*
A.61.32	161	161	161	161	161	161	161	161	P.101.52	267	267	267	267	267	267	267	267
A.62.32	85	85	85	85	85	85	85	85	P.16.9	160	160	160	160	160	160	160	160
A.63.33.1	111	111	111	111	111	111	111	111	P.19.11	178	178	178	178	178	178	178	178
A.63.33.2	125	125	125	125	125	125	125	125	P.20.11	162	162	162	162	162	162	162	162
A.64.33	102	102	102	102	102	102	102	102	P.21.12	144	144	144	144	144	144	144	144
A.65.34	116	116	116	116	116	116	116	116	P.22.12	145	145	145	145	145	145	145	145
A.69.36	125	125	125	125	125	125	125	125	P.22.12.1	7700	7700	7700	7700	7700	7700	7700	7700

(Continued)

Table A1. (Continued).

BestInt Prb.	Set1 (Tmax = 100)								BestInt Prb.	Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
	FB	NB	FB	NB	FB	NB	FB	NB		FB	NB	FB	NB	FB	NB	FB	NB
A.80.41	85	85	85	85	85	85	85	85	P.23.13	164	164	164	164	164	164	164	164
B.31.17	52	52	52	52	52	52	52	52	P.40.21	177	177	177	177	177	177	177	177
B.34.18	60	60	60	60	60	60	60	60	P.45.24	180	180	180	180	180	180	180	180
B.35.19	46	46	46	46	46	46	46	46	P.50.26	230	230	230	230	230	230	230	230
B.38.20	134	134	134	134	134	134	134	134	P.51.27	199	199	199	199	199	199	199	199
B.39.21	134	134	134	134	134	134	134	134	P.55.29	249	249	249	249	249	249	249	249
B.41.22	117	117	117	117	117	117	117	117	P.60.31	248	248	248	248	248	248	248	248
B.43.23	89	89	89	89*	89	89	89	89	P.65.34	259	259	259	259	259	259	259	259
B.44.23	95	95	95	95	95	95	95	95	P.70.36	281	281	281	281	281	281	281	281
B.45.24	98	98	98	98	98	98	98	98	P.76.39	302	302	302	302	302	302	302	302
A.32.17	58	58	58	58	58	58	58	58	B.45.24.1	88	88	88	88	88	88	88	88
A.33.18.1	75	75	75	75	75	75	75	75	B.50.26.1	110	110	110	110	110	110	110	110
A.33.18.2	99	99	99	99	99	99	99	99	B.50.26.2	109	109	109	109	109	109	109	109
A.34.18	46	46	46	46	46	46	46	46	B.51.27	50	50	50	50	50	50	50	50
A.36.19	64	64	64	64	64	64	64	64	B.52.27	145	145	145	145	145	145	145	145
A.37.1.20	82	82	82	82	82	82	82	82	B.56.29	178	178	178	178	178	178	178	178
A.37.20	60	60	60	60	60	60	60	60	B.57.30.1	54	54	54	54	54	54	54	54
A.38.20	68	68	68	68	68	68	68	68	B.57.30.2	24	24	24	24	24	24	24	24
A.39.21.1	67	67	67	67	67	67	67	67	B.63.33	64	64	64	64	64	64	64	64
A.39.21.2	124	124	124	124	124	124	124	124	B.64.33	179	179	179	179	179	179	179	179
A.44.23	90	90	90	90	90	90	90	90	B.66.34	67	67	67	67	67	67	67	67
A.45.24.1	71	71	71	71	71	71	71	71	B.67.35	116	116	116	116	116	116	116	116
A.45.24.2	57	57	57	57	57	57	57	57	B.68.35	98	98	98	98	98	98	98	98
A.46.24	73	73	73	73	73	73	73	73	B.78.40	104	104	104	104	104	104	104	104

(Continued)

Table A1. (Continued).

	Set1 (Tmax = 100)									Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB	BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB
A.48.25	62	62	62	62	62	62	62	62	G.262.132	438	415*	415*	438*	438	438	438	438
A.53.28	60	60	60	60	60	60	60	60	M.101.52	260	260	260	260*	260	260	260	260*
A.54.28	93	93	93	93	93	93	93	93	M.121.62	183*	180*	184*	163*	184*	184*	184	184*
A.55.29	89	89	89	89	89	89	89	89	M.151.77	353	353	353	353	353	353	247*	353
A.60.31	76	76	76	76	76	76	76	76	M.200.101	392*	390*	366*	343*	396*	334*	265*	366*
A.61.32	135	135	135	135	135	135	135	135	P.101.52	221	221	221	221*	221	221	221	221
A.62.32	85	85	85	85	85	85	85	85	P.16.9	131	131	131	131	131	131	131	131
A.63.33.1	85	85	85	85	85	85	85	85	P.19.11	120	120	120	120	120	120	120	120
A.63.33.2	103	103	103	103	103	103	103	103	P.20.11	126	126	126	126	126	126	126	126
A.64.33	102	102	102	102	102	102	102	102	P.21.12	121	121	121	121	121	121	121	121
A.65.34	102	102	102	102	102	102	102	102	P.22.12	131	131	131	131	131	131	131	131
A.69.36	114	114	114	114	114	114	114	114	P.22.12.1	6100	6100	6100	6100	6100	6100	6100	6100
A.80.41	84	84	84	84	84	84	84	84	P.23.13	122	122	122	122	122	122	122	122
B.31.17	51	51	51	51	51	51	51	51	P.40.21	169	169	169	169	169	169	169	169
B.34.18	41	41	41	41	41	41	41	41	P.45.24	171	171	171	171	171	171	171	171
B.35.19	46	46	46	46	46	46	46	46	P.50.26	196	196	196	196	196	196	196	196
B.38.20	91	91	91	91	91	91	91	91	P.51.27	172	172	172	172	172	172	172	172
B.39.21	85	85	85	85	85	85	85	85	P.55.29	228	228	228	228	228	228	228	228
B.41.22	74	74	74	74	74	74	74	74	P.60.31	232	232	232	232	232	232	232	232
B.43.23	59	59	59	59	59	59	59	59	P.65.34	214	214	214	214	214	214	214	214
B.44.23	85	85	85	85	85	85	85	85	P.70.36	235	235	235	235	235	235	235	235
B.45.24	62	62	62	62	62	62	62	62	P.76.39	235	235	235	235	235	235	235	235
A.32.17	115	115	115	115	115	115	115	115	B.45.24.1	237	237	237	237	237	237	237	237
A.33.18.1	167	167	167	167	167	167	167	167	B.50.26.1	242	240*	242	240*	242	242*	242	242*

(Continued)

Table A1. (Continued).

	Set1 (Tmax = 100)									Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB	BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB
A.33.18.2	220	220	220	220	220	220	220	220	B.50.26.2	277	277	277	277	277	277	277	277
A.34.18	143	143	143	143	143	143	143	143	B.51.27	226	226*	226	226*	226	226*	226	226*
A.36.19	162	162	162	162	162	162	162	162	B.52.27	224*	224*	224*	224*	224*	224*	224*	224*
A.37.1.20	177	177	177	177	177	177	177	177	B.56.29	275*	272*	275*	272*	275	275*	275	275*
A.37.20	208	208	208	208	208	208	208	208	B.57.30.1	197*	190*	197*	190*	198*	197*	198	190*
A.38.20	187	187	187	187	187	187	187	187	B.57.30.2	243	243*	243	243*	243	243*	243	243*
A.39.21.1	171	171	171	171	171	171	171	171	B.63.33	227*	227*	225*	204*	225*	224*	225*	207*
A.39.21.2	212	212	212	212	212	212	212	212	B.64.33	404*	404*	410	410*	410	404*	410	410*
A.44.23	200	200	200	200	200	200	200	200	B.66.34	292*	292*	292*	292*	292*	292*	292	292*
A.45.24.1	188	188	188	188	188	188	188	188	B.67.35	341*	336*	347	347*	347*	347*	347	337*
A.45.24.2	192	192	192	192	192	192	192	192	B.68.35	283*	284*	281*	238*	284*	237*	284*	278*
A.46.24	209	209	209	209	209	209	209	209	B.78.40	314*	300*	314*	299*	313*	314*	314*	300*
A.48.25	177	177	177	177	177	177	177	177	G.262.132	907*	942*	711*	862*	1049*	878*	1006*	925*
A.53.28	194	194	194	194	194	194	194	194	M.101.52	630*	580*	630*	630*	640*	580*	640*	580*
A.54.28	222	222	222	222	222	222	222	222	M.121.62	346*	345*	353*	338*	351*	333*	354*	340*
A.55.29	258	258	258	258	258	258	258	258	M.151.77	774	774	774	774	774*	774	774	774
A.60.31	257	257	257	257	257	257	257	257	M.200.101	887*	938*	774*	933*	918*	943*	863*	943*
A.61.32	321	321	321	321	321	321	321	321	P.101.52	528*	528	528	528	528*	528	528	528
A.62.32	222	222	222	222	222	222	222	222	P.16.9	179	179	179	179	179	179	179	179
A.63.33.1	220	220	220	220	220	220	220	220	P.19.11	218	218	218	218	218	218	218	218
A.63.33.2	311	311	311	311	311	311	311	311	P.20.11	211	211	211	211	211	211	211	211
A.64.33	264	264*	264	264*	264	264*	264	264	P.21.12	203	203	203	203	203	203	203	203
A.65.34	206	206	206	206*	206	206	206	206	P.22.12	207	207	207	207	207	207	207	207
A.69.36	275	275	275	275	275	275	275	275	P.22.12.1	14,600	14,600	14,600	14,600	14,600	14,600	14,600	14,600

(Continued)

Table A1. (Continued).

	Set1 (Tmax = 100)									Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB	BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB
A.80.41	228	228	228	228	228	228	228	228	P.23.13	213	213	213	213	213	213	213	213
B.31.17	233	233	233	233	233	233	233	233	P.40.21	328	328	328	328	328	328	328	328
B.34.18	261	261	261	261	261	261	261	261	P.45.24	345	345	345	345	345	345	345	345
B.35.19	86	86	86	86*	86	86	86	86	P.50.26	448	448	448	448	448	448	448	448
B.38.20	221	221	221	221	221	221	221	221	P.51.27	374	374	374	374	374	374	374	374
B.39.21	220	220	220	220	220	220	220	220	P.55.29	498	498	498	498	498	498	498	498
B.41.22	203	203*	203	203*	203	203	203	203	P.60.31	494	494	494	494	494	494	494	494
B.43.23	225	225	225	225*	225	225	225	224*	P.65.34	478	478	478	478	478	478	478	478
B.44.23	240*	240*	240	240*	240	236*	240	236*	P.70.36	512	512	512	512	512	512	512	512
B.45.24	182	182*	182	182*	182	182*	182	182*	P.76.39	542	542	542	542	542	542	542	542
A.32.17	115	115	115	115	115	115	115	115	B.45.24.1	191	191	191	191	191	191	191	191
A.33.18.1	128	128	128	128	128	128	128	128	B.50.26.1	212	212	212	212	212	212	212	212
A.33.18.2	168	168	168	168	168	168	168	168	B.50.26.2	200	200	200	200	200	200	200	200
A.34.18	121	121	121	121	121	121	121	121	B.51.27	179	179	179	179	179	179	179	179
A.36.19	125	125	125	125	125	125	125	125	B.52.27	192	192	192	192	192	192*	192*	192
A.37.1.20	145	145	145	145	145	145	145	145	B.56.29	223	220*	223	223	223	223	223	223
A.37.20	189	189	189	189	189	189	189	189	B.57.30.1	156	156*	156*	156*	156	156*	156	151*
A.38.20	150	150	150	150	150	150	150	150	B.57.30.2	156	156	156	156*	156	156	156	156*
A.39.21.1	156	156	156	156	156	156	156	156	B.63.33	190	190*	190	190*	190	190*	190	190*
A.39.21.2	186	186	186	186	186	186	186	186	B.64.33	360	360	360	360	360	360	360	360
A.44.23	154	154	154	154	154	154	154	154	B.66.34	202	202	202	202*	202	202	202	202
A.45.24.1	159	159	159	159	159	159	159	159	B.67.35	266	266	266	266*	266	266	266	263*
A.45.24.2	140	140	140	140	140	140	140	140	B.68.35	259*	259*	259*	259*	259*	259*	259*	249*
A.46.24	181	181	181	181	181	181	181	181	B.78.40	218*	218*	218*	207*	218	218*	218	207*

(*Continued*)

Table A1. (Continued).

BestInt Prb.	Set1 (Tmax = 100)								BestInt Prb.	Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
	FB	NB	FB	NB	FB	NB	FB	NB		FB	NB	FB	NB	FB	NB	FB	NB
A.48.25	167	167	167	167	167	167	167	167	G.262.132	814*	859*	638*	791*	867*	849*	541*	700*
A.53.28	149	149	149	149	149	149	149	149	M.101.52	560*	570*	570*	570*	570*	570*	570*	570*
A.54.28	202	202	202	202	202	202	202	202	M.121.62	274*	278*	277*	278*	278*	274*	278*	275*
A.55.29	216	216	216	216	216	216	216	216	M.151.77	672	672	672	662*	662*	672	651*	672
A.60.31	196	196	196	196	196	196	196	196	M.200.101	851*	851	758*	851*	841*	851	752*	851*
A.61.32	274	274	274	274	274	274	274	274	P.101.52	454	454	454	454	454*	454	454	454
A.62.32	205	205	205	205	205	205	205	205	P.16.9	150	150	150	150	150	150	150	150
A.63.33.1	200	200	200	200	200	200	200	200	P.19.11	158	158	158	158	158	158	158	158
A.63.33.2	265	265	265	265	265	265	265	265	P.20.11	151	151	151	151	151	151	151	151
A.64.33	219	219	219	219	219	219	219	219	P.21.12	146	146	146	146	146	146	146	146
A.65.34	199	199	199	199	199	199	199	199	P.22.12	172	172	172	172	172	172	172	172
A.69.36	247	247	247	247	247	247	247	247	P.22.12.1	12300	12300	12300	12300	12300	12300	12300	12300
A.80.41	194	194	194	191*	194	194	194	194	P.23.13	148	148	148	148	148	148	148	148
B.31.17	169	169	169	169	169	169	169	169	P.40.21	302	302	302	302	302	302	302	302
B.34.18	221	221	221	221	221	221	221	221	P.45.24	309	309	309	309	309	309	309	309
B.35.19	67	67	67	67	67	67	67	67	P.50.26	367	367	367	367	367	367	367	367
B.38.20	178	178	178	178	178	178	178	178	P.51.27	327	327	327	327	327	327	327	327
B.39.21	153	153	153	153	153	153	153	153	P.55.29	418	418	418	418	418	418	418	418
B.41.22	162	162	162	162	162	162	162	162	P.60.31	415	415	415	415	415	415	415	415
B.43.23	170	170	170	170	170	170	170	170	P.65.34	394	394	394	394	394	394	394	394
B.44.23	217	217	217	217	217	217	217	217	P.70.36	424	424	424	424	424	424	424	424
B.45.24	141	141	141	141	141	141	141	141	P.76.39	424	424	424	424	424	424	424	424
A.32.17	198	198	198	198	198	198	198	198	B.45.24.1	307	307	307	307	307	307	307	307
A.33.18.1	249	249	249	249	249	249	249	249	B.50.26.1	363*	355*	363	342	363*	363*	363	342*

(Continued)

Table A1. (Continued).

	Set1 (Tmax = 100)									Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB	BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB
A.33.18.2	290	290	290	290	290	290	290	290	B.50.26.2	446	446	446	446	446	446	446	446
A.34.18	226	226	226	226	226	226	226	226	B.51.27	308*	295*	308	308*	308*	295*	308*	308*
A.36.19	236	236	236	236	236	236	236	236	B.52.27	350	350*	350	350*	350*	350*	350	350*
A.37.1.20	241	241	241	241	241	241	241	241	B.56.29	365	365*	365*	365*	365*	365*	365*	365*
A.37.20	304	304	304	304	304	304	304	304	B.57.30.1	319*	319*	322*	319*	319*	319*	322*	319*
A.38.20	266	266	266	266	266	266	266	266	B.57.30.2	456	456*	456	450*	456	456*	456	456*
A.39.21.1	257	257	257	257	257	257	257	257	B.63.33	409*	396*	409	396*	409	407*	409	409*
A.39.21.2	321	321	321	321	321	321	321	321	B.64.33	509	509	509	509*	509	509*	509	509
A.44.23	293	293	293	293	293	293	293	293	B.66.34	427	427*	427	427*	427	427*	427	427*
A.45.24.1	267	267	267	267	267	267	267	267	B.67.35	473*	467*	477	476*	477	473*	477	473*
A.45.24.2	301	301	301	301	301	301	301	301	B.68.35	474*	474*	474	474*	474*	474*	474	474*
A.46.24	304	304	304	304	304	304	304	304	B.78.40	484*	484*	484*	462*	476*	451*	476*	475*
A.48.25	298	298	298	298	298	298	298	298	G.262.132	1386*	1400*	1188*	1340*	1413*	1350*	1454*	1336*
A.53.28	306	306	306	306	306	306	306	306	M.101.52	960*	940*	960*	960*	930*	820*	960*	900*
A.54.28	353	353	353	353	353	353	353	353	M.121.62	537*	479*	545*	475*	545*	483*	542*	535*
A.55.29	373	373	373	373	373	373	373	373	M.151.77	1077	1077	1077	1077	1077	1077	1077	1077
A.60.31	406	406	406	406	406	406	406	406	M.200.101	1317*	1321*	1212*	1329*	1296*	1329*	1319*	1326*
A.61.32	439	439	439	439	439	439	439	439	P.101.52	743	743	743	743	743	743	743	743
A.62.32	311	311	311	311	311	311	311	311	P.16.9	179	179	179	179	179	179	179	179
A.63.33.1	378	378	378	378	378	378	378	378	P.19.11	218	218	218	218	218	218	218	218
A.63.33.2	414	414	414	414	414	414	414	414	P.20.11	211	211	211	211	211	211	211	211
A.64.33	389	389	389	389	389	389	389	389	P.21.12	203	203	203	203	203	203	203	203
A.65.34	346	346	346	346	346	346	346	346	P.22.12	207	207	207	207	207	207	207	207
A.69.36	397	397	397	397	397	397	397	397	P.22.12.1	15800	15800	15800	15800	15800	15800	15800	15800

(Continued)

Table A1. (Continued).

BestInt Prb.	Set1 (Tmax = 100)								BestInt Prb.	Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
	FB	NB	FB	NB	FB	NB	FB	NB		FB	NB	FB	NB	FB	NB	FB	NB
A.80.41	361	361	361	361	361	361	361	361	P.23.13	213	213	213	213	213	213	213	213
B.31.17	251	251	251	251	251	251	251	251	P.40.21	396	396	396	396	396	396	396	396
B.34.18	352	352	352	352	352	352	352	352	P.45.24	452	452	452	452	452	452	452	452
B.35.19	228	228	228	228	228	228	228	228	P.50.26	563	563	563	563	563	563	563	563
B.38.20	300	300	300	300	300	300	300	300	P.51.27	499	499	499	499	499	499	499	499
B.39.21	297	297	297	297	297	297	297	297	P.55.29	629	629	629	629	629	629	629	629
B.41.22	306	306	306	306	306	306	306	306	P.60.31	656	656	656	656	656	656	656	656
B.43.23	342	342	342	342	342	342	342	342	P.65.34	642	642	642	642	642	642	642	642
B.44.23	419	419	419	419	419	419	419	419	P.70.36	681	681	681	681	681	681	681	681
B.45.24	286	286	286	286	286	286	286	286	P.76.39	709	709	709	709	709	709	709	709
A.32.17	175	175	175	175	175	175	175	175	B.45.24.1	235	235	235	235	235	235	235	235
A.33.18.1	194	194	194	194	194	194	194	194	B.50.26.1	301	301	301	301	301	301	301	301
A.33.18.2	247	247	247	247	247	247	247	247	B.50.26.2	331	331	331	331	331	331	331	331
A.34.18	192	192	192	192	192	192	192	192	B.51.27	244	244	244	244	244	244	244	244
A.36.19	183	183	183	183	183	183	183	183	B.52.27	294	294	294	294	294	294	294	294
A.37.1.20	208	208	208	208	208	208	208	208	B.56.29	280	280	280	280*	280*	280	279*	280
A.37.20	280	280	280	280	280	280	280	280	B.57.30.1	263	261*	263	261*	263	263*	263	261*
A.38.20	209	209	209	209	209	209	209	209	B.57.30.2	328	328	328	328	328	328	328	328
A.39.21.1	216	216	216	216	216	216	216	216	B.63.33	336	336*	336	336*	336	336*	336	336*
A.39.21.2	265	265	265	265	265	265	265	265	B.64.33	427	427	427	427	427	427	427	427
A.44.23	230	230	230	230	230	230	230	230	B.66.34	294	294	294	294	294	294	294	294
A.45.24.1	230	230	230	230	230	230	230	230	B.67.35	381	381	381	381	381	381	381	381
A.45.24.2	240	240	240	240	240	240	240	240	B.68.35	401	401	401	401	401	401	401	401
A.46.24	252	252	252	252	252	252	252	252	B.78.40	363	363*	363*	362*	363*	363*	363*	363*

(Continued)

Table A1. (Continued).

	Set1 (Tmax = 100)									Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB	BestInt Prb.	FB	NB	FB	NB	FB	NB	FB	NB
A.48.25	265	265	265	265	265	265	265	265	G.262.132	1180*	1031*	618*	893*	1168*	1185*	864*	1177*
A.53.28	245	245	245	245	245	245	245	245	M.101.52	830	830*	830*	830*	830*	830*	830*	830*
A.54.28	283	283	283	283	283	283	283	283	M.121.62	460*	461*	466*	372*	468*	461*	463*	449*
A.55.29	334	334	334	334	334	334	334	334	M.151.77	909	909	909*	909	909*	909	892*	909
A.60.31	325	325	325	325	325	325	325	325	M.200.101	1184*	1183*	1147*	1180*	1176*	1184	1045*	1184*
A.61.32	359	359	359	359	359	359	359	359	P.101.52	628	628	628	628	628*	628	628	628
A.62.32	285	285	285	285	285	285	285	285	P.16.9	150	150	150	150	150	150	150	150
A.63.33.1	322	322	322	322	322	322	322	322	P.19.11	158	158	158	158	158	158	158	158
A.63.33.2	366	366	366	366	366	366	366	366	P.20.11	151	151	151	151	151	151	151	151
A.64.33	318	318	318	318	318	318	318	318	P.21.12	146	146	146	146	146	146	146	146
A.65.34	327	327	327	327	327	327	327	327	P.22.12	172	172	172	172	172	172	172	172
A.69.36	348	348	348	348	348	348	348	348	P.22.12.1	12300	12300	12300	12300	12300	12300	12300	12300
A.80.41	293	293	293	293*	293	293	293	293	P.23.13	148	148	148	148	148	148	148	148
B.31.17	184	184	184	184	184	184	184	184	P.40.21	314	314	314	314	314	314	314	314
B.34.18	280	280	280	280	280	280	280	280	P.45.24	351	351	351	351	351	351	351	351
B.35.19	204	204	204	204	204	204	204	204	P.50.26	418	418	418	418	418	418	418	418
B.38.20	258	258	258	258	258	258	258	258	P.51.27	377	377	377	377	377	377	377	377
B.39.21	206	206	206	206	206	206	206	206	P.55.29	471	471	471	471	471	471	471	471
B.41.22	208	208	208	208	208	208	208	208	P.60.31	496	496	496	496	496	496	496	496
B.43.23	254	254	254	254	254	254	254	254	P.65.34	523	523	523	523	523	523	523	523
B.44.23	347	347	347	347	347	347	347	347	P.70.36	560	560	560	560	560	560	560	560
B.45.24	200	200	200	200	200	200	200	200	P.76.39	567	567	567	567	567	567	567	567
A.32.17	253	253	253	253	253	253	253	253	B.45.24.1	359	359	359	359	359	359	359	359
A.33.18.1	281	281	281	281	281	281	281	281	B.50.26.1	419	419	419	419	419	419	419	419

(Continued)

Table A1. (Continued).

BestInt Prb.	Set1 (Tmax = 100)								BestInt Prb.	Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
	FB	NB	FB	NB	FB	NB	FB	NB		FB	NB	FB	NB	FB	NB	FB	NB
A.33.18.2	333	333	333	333	333	333	333	333	B.50.26.2	494	494	494	494	494	494	494	494
A.34.18	279	279	279	279	279	279	279	279	B.51.27	406	406*	406	406*	406	406*	406*	406*
A.36.19	259	259	259	259	259	259	259	259	B.52.27	399	399	399	399	399	399	399	399
A.37.1.20	284	284	284	284	284	284	284	284	B.56.29	400	400	400	400	400	400	400	400
A.37.20	361	361	361	361	361	361	361	361	B.57.30.1	459	459*	459*	459*	459*	455*	459*	455*
A.38.20	312	312	312	312	312	312	312	312	B.57.30.2	527	527	527	527	527	527	527	527
A.39.21.1	306	306	306	306	306	306	306	306	B.63.33	550	550*	550	550*	550	550	550	549*
A.39.21.2	371	371	371	371	371	371	371	371	B.64.33	546	546	546	546	546	546	546	546
A.44.23	334	334	334	334	334	334	334	334	B.66.34	490	490	490	490	490	490	490	490
A.45.24.1	337	337	337	337	337	337	337	337	B.67.35	536	536	536	536	536	536	536	536
A.45.24.2	357	357	357	357	357	357	357	357	B.68.35	522	522	522	522	522	522	522	522
A.46.24	361	361	361	361	361	361	361	361	B.78.40	605	605*	605	604*	605	589*	605	605*
A.48.25	377	377	377	377	377	377	377	377	G.262.132	1871*	2081*	1137*	1754*	1751*	1658*	1982*	1719*
A.53.28	397	397	397	397	397	397	397	397	M.101.52	1170*	1060*	1160*	1150*	1170*	1060*	1150*	1130*
A.54.28	422	422	422	422	422	422	422	422	M.121.62	772*	772*	772*	770*	769*	621*	765*	769*
A.55.29	488	488	488	488	488	488	488	488	M.151.77	1296	1296	1296	1296	1296	1296	1296	1296
A.60.31	512	512	512	512	512	512	512	512	M.200.101	1623*	1654*	1630*	1648*	1610*	1654*	1654*	1631*
A.61.32	526	526	526	526	526	526	526	526	P.101.52	891	891	891	891	891	891	891	891
A.62.32	418	418	418	418	418	418	418	418	P.16.9	179	179	179	179	179	179	179	179
A.63.33.1	497	497	497	497	497	497	497	497	P.19.11	218	218	218	218	218	218	218	218
A.63.33.2	508	508	508	508	508	508	508	508	P.20.11	211	211	211	211	211	211	211	211
A.64.33	480	480	480	480	480	480	480	480	P.21.12	203	203	203	203	203	203	203	203
A.65.34	468	468	468	468	468	468	468	468	P.22.12	207	207	207	207	207	207	207	207
A.69.36	505	505	505	505	505	505	505	505	P.22.12.1	15800	15800	15800	15800	15800	15800	15800	15800

(Continued)

Table A1. (Continued).

BestInt Prb.	Set1 (Tmax = 100)								BestInt Prb.	Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
	FB	NB	FB	NB	FB	NB	FB	NB		FB	NB	FB	NB	FB	NB	FB	NB
A.80.41	460	460	460	460	460	460	460	460	P.23.13	213	213	213	213	213	213	213	213
B.31.17	251	251	251	251	251	251	251	251	P.40.21	396	396	396	396	396	396	396	396
B.34.18	352	352	352	352	352	352	352	352	P.45.24	452	452	452	452	452	452	452	452
B.35.19	287	287	287	287	287	287	287	287	P.50.26	563	563	563	563	563	563	563	563
B.38.20	339	339	339	339	339	339	339	339	P.51.27	509	509	509	509	509	509	509	509
B.39.21	297	297	297	297	297	297	297	297	P.55.29	637	637	637	637	637	637	637	637
B.41.22	343	343	343	343	343	343	343	343	P.60.31	691	691	691	691	691	691	691	691
B.43.23	342	342	342	342	342	342	342	342	P.65.34	734	734	734	734	734	734	734	734
B.44.23	419	419	419	419	419	419	419	419	P.70.36	779	779	779	779	779	779	779	779
B.45.24	334	334	334	334	334	334	334	334	P.76.39	831	831	831	831	831	831	831	831
A.32.17	207	207	207	207	207	207	207	207	B.45.24.1	257	257	257	257	257	257	257	257
A.33.18.1	194	194	194	194	194	194	194	194	B.50.26.1	317	317	317	317	317	317	317	317
A.33.18.2	259	259	259	259	259	259	259	259	B.50.26.2	372	372	372	372	372	372	372	372
A.34.18	229	229	229	229	229	229	229	229	B.51.27	327	327	327	327	327	327	327	327
A.36.19	194	194	194	194	194	194	194	194	B.52.27	302	302	302	302	302	302	302	302
A.37.1.20	228	228	228	228	228	228	228	228	B.56.29	314	314	314	314	314	314	314	314
A.37.20	301	301	301	301	301	301	301	301	B.57.30.1	365	365	365	365	365	365	365	365
A.38.20	235	235	235	235	235	235	235	235	B.57.30.2	358	358	358	358	358	358	358	358
A.39.21.1	222	222	222	222	222	222	222	222	B.63.33	428	428	428	428	428	428	428	428
A.39.21.2	293	293	293	293	293	293	293	293	B.64.33	464	464	464	464	464	464	464	464
A.44.23	260	260	260	260	260	260	260	260	B.66.34	339	339	339	339	339	339	339	339
A.45.24.1	273	273	273	273	273	273	273	273	B.67.35	439	439	439	439	439	439	439	439
A.45.24.2	278	278	278	278	278	278	278	278	B.68.35	422	422	422	422	422	422	422	422
A.46.24	276	276	276	276	276	276	276	276	B.78.40	443	443	443	443	443	443	443	443

(Continued)

Table A1. (Continued).

BestInt Prb.	Set1 (Tmax = 100)								BestInt Prb.	Set1 (Tmax = 100)							
	SB				TB					SB				TB			
	C_C		N_N		C_C		N_N			C_C		N_N		C_C		N_N	
	FB	NB	FB	NB	FB	NB	FB	NB		FB	NB	FB	NB	FB	NB	FB	NB
A.48.25	308	308	308	308	308	308	308	308	G.262.132	1637*	1383*	1241*	1426*	1472*	1824*	1284*	1640*
A.53.28	293	293	293	293	293	293	293	293	M.101.52	930	930	930	930	930	930	930	930
A.54.28	330	330	330	330	330	330	330	330	M.121.62	639	639*	639	639*	639*	639	639	639*
A.55.29	402	402	402	402	402	402	402	402	M.151.77	1064	1064	1064	1064	1064	1064	1064	1064
A.60.31	400	400	400	400	400	400	400	400	M.200.101	1431	1431	1431	1431	1431	1431	1396*	1431
A.61.32	422	422	422	422	422	422	422	422	P.101.52	712	712	712	712	712	712	712	712
A.62.32	368	368	368	368	368	368	368	368	P.16.9	150	150	150	150	150	150	150	150
A.63.33.1	396	396	396	396	396	396	396	396	P.19.11	158	158	158	158	158	158	158	158
A.63.33.2	448	448	448	448	448	448	448	448	P.20.11	151	151	151	151	151	151	151	151
A.64.33	385	385	385	385	385	385	385	385	P.21.12	146	146	146	146	146	146	146	146
A.65.34	402	402	402	402	402	402	402	402	P.22.12	172	172	172	172	172	172	172	172
A.69.36	427	427	427	427	427	427	427	427	P.22.12.1	12300	12300	12300	12300	12300	12300	12300	12300
A.80.41	393	393	393	393	393	393	393	393	P.23.13	148	148	148	148	148	148	148	148
B.31.17	184	184	184	184	184	184	184	184	P.40.21	314	314	314	314	314	314	314	314
B.34.18	280	280	280	280	280	280	280	280	P.45.24	351	351	351	351	351	351	351	351
B.35.19	235	235	235	235	235	235	235	235	P.50.26	418	418	418	418	418	418	418	418
B.38.20	267	267	267	267	267	267	267	267	P.51.27	377	377	377	377	377	377	377	377
B.39.21	206	206	206	206	206	206	206	206	P.55.29	471	471	471	471	471	471	471	471
B.41.22	243	243	243	243	243	243	243	243	P.60.31	496	496	496	496	496	496	496	496
B.43.23	254	254	254	254	254	254	254	254	P.65.34	539	539	539	539	539	539	539	539
B.44.23	347	347	347	347	347	347	347	347	P.70.36	582	582	582	582	582	582	582	582
B.45.24	234	234	234	234	234	234	234	234	P.76.39	603	603	603	603	603	603	603	603