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New integer programming formulation for multiple traveling repairmen problem

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Abstract

The multiple traveling repairman problem (kTRP) is a generalization of the traveling repairman problem which is also known as the minimum latency problem and the deliveryman problem. In these problems, waiting time or latency of a customer is defined as the time passed from the beginning of the travel until this customer's service completed. The objective is to find a Hamiltonian Tour or a Hamiltonian Path that minimizes the total waiting time of customers so that each customer is visited by one of the repairmen. In this paper, we propose a new mixed integer linear programming formulation for the multiple traveling repairman problem where each repairman starts from the depot and finishes the journey at a given node. In order to see the performance of the proposed formulation against existing formulations, we conduct computational analysis by solving benchmark instances appeared in the literature. Computational results show that proposed model is extremely effective than the others in terms of CPU times.

Keywords: Traveling repairmen; traveling salesman; minimum latency; routing; modeling

1. Introduction

The Traveling Salesman Problem (TSP) is the basis of the routing problems. The Traveling Repairman Problem (TRP) which is also named as the minimum latency problem, the cumulative traveling salesman problem or the traveling deliveryman problem (Silva et al., 2012) is a special type of routing problem. In these problems, waiting

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time or latency of a customer is defined as the time passed from the beginning of the travel until this customer's service completed is. The Multiple Traveling Repairman Problem (kTRP) is a generalization of the TRP hence the Minimum Latency Problem finds *k* tours or paths, each starting at the depot and covering all the nodes while minimizing total waiting time (latency). Applications of this problem can be found in home delivery of pizzas, emergency aid logistics, routing automated guided vehicles in a flexible manufacturing system, school bus routing and minimizing average flow time for jobs of scheduling machines (Fischetti et al., 1993). It has been shown kTRP is NP-hard (Tsitsiklis, 1992; Archer and Williamson, 2003). Thus, solution strategies for kTRP are concentrated on the exact solution procedures by Picard and Queyranne (1978), Lucena (1990), Simchl-Levi and Berman (1991), Bianco et al. (1993), Fischetti et al. (1993), Eijl (1995), Wu et al. (2004) and/or heuristics by Blum et al. (1994), Goemans and Kleinberg (1998), Ausiello et al. (2000), Arora and Karakostas (2003), Chaudhuri et al. (2003), Nagarajan and Ravi (2008), Salehipour et al. (2011), Ngueveu et al. (2010) and Dewilde et al. (2010).

There exist a few formulations for finding the optimal solution of the problem directly in the literature. Sarubbi et al. (2008) applied for the minimum latency problem the model proposed by Picard and Queyranne (1978) for the time-dependent the traveling salesman problem. Kara et al. (2008) developed a mixed integer linear programming formulation for the minimum latency problem. Mendez-Diaz et al. (2008) suggested an integer programming formulation for the traveling deliveryman problem. Angel-Bello et al. (2013) developed two integer programming formulations for TRP. They reviewed existing formulations and conducted a comparative computational analysis of the formulations. They conclude that one of the new formulations named as model A is superior to the others. The emerging developments in the information technology allow us to find optimal solution of some routing problems directly by using a suitable software and user friendly formulations. Recently, Kara and Derya (2015) found the shortest tour time of 400-node traveling salesman problem with time windows within seconds using CPLEX 12.5. Those developments motivate us to develop new mathematical models for the k-traveling repairman problem.

In this paper, we adapt model A of Angel-Bello et al. formulation to the k-traveling repairman problem and we compute the performance of this formulation against the existing k-traveling repairman formulations. The main contribution of this paper is to present a new polynomial size integer programming formulation for the k-traveling repairman problem that can be used to find optimal solutions of the real life problems.

In Section 2, we present the definition and application of the k-traveling repairman problem and we investigate the existing formulations of the k-traveling repairman problem in the literature. We propose a new mathematical model for the k-traveling repairman problem in Section 3. We conduct computational analysis of the proposed model against existing models and summarize the results in Section 4. Concluding remarks are outlined in Section 5.

2. Problem identification and existing formulations

Given a network G = (V, A) where V= {1, 2… n} is the node set of the customers, {0} is the depot and {n} is the terminal node. A = {(i, j): i, j $\in$ V, i $\neq$ j} is the set of arcs. d_{ij} is the time of the travel from the node *i* to the node *j*. *k* is the number of identical travelers. x_{ij} is the decision variable. $x_{ij} = 1$ if the arc (i, j) is on the repairman, and zero otherwise.

With those given above, we define k-TRP as:

- Each node (customer) is served exactly by one traveler,
- Each route starts from the depot and ends at the terminal node,
- The objective is to find a set of k traveler routes of minimum total time passed until the all customers served.

The mathematical models of the k-repairmen problem in the literature are explained in this Section. Kara et al. (2008), by defining additional arc based decision variables y_{ij} as;

y_{ij} = if the arc (i,j) is on the path then this variable shows the sequence of node j from the end, and zero otherwise. Their formulation has O(n^2) binary variables and O(n^2) constraints. We named this formulation as M1 and used in computational analysis.

Luo et al. (2014), defined node based decision variables u_{ik} as the arriving time of vehicle k to node i and then presented an integer programming formulation. This model has exponential number of constraints, thus it is insufficient for direct use with an optimizer.

Fishetti et al. (1993) presented an integer programming formulation for TRP and defined additional arc based decision variables z_{ij} as;

$$z_{ij} = \begin{cases} n-m+1, \text{ if arc(i,j) is on the position m} \\ 0, \text{ otherwise} \end{cases}$$

This formulation was developed for the minimum latency problem originally. Onder (2015) transformed this formulation to k-traveling repairmen by adding two new constraints, so repairmen exit from the depot and finish at the depot. In our study, this formulation is used that it gives a path finishes at a terminal node. This formulation has n^2 binary variables, nonnegative variables and n^2+4n constraints. We named this formulation as YM1 and used in computational analysis.

3. Proposed formulation

In proposed formulation an artificial position for starting node (depot) is defined as a different notation. This artificial depot is represented by n+1.

$$x_i^m = \begin{cases} 1, \text{ if node i is on the position m} \\ 0, \text{ otherwise} \end{cases}$$

$$y_{ij}^m = \begin{cases} 1, \text{ if node i is on the position m and node j is on the position m+1} \\ 0, \text{ otherwise} \end{cases}$$

The model is given that:

$$Min \sum_{m=1}^{n}\sum_{i=1}^{n} m d_{0i} y_{in+1}^m + \sum_{m=1}^{n-1}\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n} (m-1) d_{ij} y_{ij}^m \tag{1}$$

subject to

$$\sum_{m=1}^{n} x_i^m = 1 \qquad i = 1,2...n \tag{2}$$

$$\sum_{i=1}^{n} x_i^m \leq k \qquad m = 1,2...n \tag{3}$$

$$\sum_{i=1}^{n} x_i^1 = k \tag{4}$$

$$\sum_{\substack{j=1 \\ j\neq i}}^{n+1} y_{ij}^m = x_i^m \qquad i = 1,2...n \qquad m = 1,2...n-1 \tag{5}$$

$$y_{in+1}^n = x_i^n \qquad i = 1,2...n \tag{6}$$

$$\sum_{\substack{j=1 \\ j\neq i}}^{n} y_{ji}^m = x_i^{m+1} \qquad i = 1,2...n \qquad m = 1,2...n-1 \tag{7}$$

$$x_i^m \in \{0,1\} \qquad i,m = 1,2...n \tag{8}$$

$$y_{ij}^m \geq 0 \qquad i,j = 1,2...n \qquad m = 1,2...n-1 \qquad j \neq i \tag{9}$$

While developing this formulation, the definitions of the decision variables are used as original form for the minimum latency problem. The objective function, given in (1), ensures the minimum total waiting time. Constraint (2) makes that each node is at one position. Constraint (3) ensures that in each position there must be nodes at most the number of repairmen. Constraint (4) provides that k-repairmen exit from the depot. Constraint (6) is about the relation between decision variables y_{ij}^{m} and x_i^m to determine the last node visited. Constraint (5) ensures that only one arc exits from position m and constraint (7) ensures that only one arc arrives in position m+1.

The objective function is rewritten as meaning and structurally, so the structure and the meaning of objective function of the minimum latency problem formulation developed by Angel-Bello et al. are changed completely and our objective function is entirely new. In this way, we obtain a new formulation that can solve multiple traveling repairmen problem. This formulation has n^2 binary variables and $2n^2+n+1$ constraints, so our formulation is polynomial size that can be used directly by an optimizer. We named this formulation as YM2 and used in computational analysis.

4. Computational analysis

In order to see the performance of the proposed formulation, we conduct a computational analysis. We summarize the results in this section.

Comparisons are made between the formulations of M1, YM1 and YM2 on a set of instances taken from the literature (Salehipour et al., 2011). These sets include 10 and 20 node problems. 20 symmetric instances are prepared for these sets. Computational comparisons are focused on CPU times. All problems are solved with CPLEX 12.6.0.0 by using Intel Core Quad CPU 2.66 GHz and 2 GB RAM computer. The upper time limit is defined as 7200 seconds for all computations. Solution values obtained in time limit are used to calculate mean and standard deviation. If k is equal to 1, all the formulations can find the optimal solutions for one repairman. 20 benchmark instances for 10 node problems are solved for k=2 by each formulation. CPU times and optimal values are given in Table 1.

Table 1. CPU times and optimal values of 10 node problems for k=2

Problem	Optimal value	M1 CPU (Sec)	YM1 CPU (Sec)	YM2 CPU (Sec)	Problem	Optimal value	M1 CPU (Sec)	YM1 CPU (Sec)	YM2 CPU (Sec)
1	828	0.13	0.16	0.11	11	767	0.10	0.28	0.02
2	872	0.17	0.22	0.02	12	712	0.14	0.24	0.07
3	846	0.21	0.44	0.22	13	812	0.34	0.63	0.11
4	821	0.28	0.28	0.22	14	738	0.27	0.31	0.18
5	596	0.20	0.24	0.02	15	690	0.21	0.20	0.25
6	939	0.31	0.28	0.19	16	676	0.47	0.35	0.09
7	741	0.24	0.29	0.02	17	691	0.11	0.22	0.23
8	681	0.28	0.30	0.14	18	661	0.27	0.20	0.18
9	820	0.24	0.35	0.20	19	829	0.21	0.32	0.15
10	756	0.27	0.60	0.18	20	687	0.30	0.27	0.13
Mean						-	0.24	0.31	0.14
Standard deviation						-	0.09	0.12	0.08

According to CPU times, YM2 is faster than the other formulations for 10 node problems and k=2. Mean and standard deviation of CPU of the YM2 are considerably smaller than the others. All of these problems can be solved in time limit 7200 seconds by all formulations. Also, 20 benchmark instances for 20 node problems are solved for k=2 by each formulation. CPU times and optimal values are given in Table 2.

Table 2. CPU times and optimal values of 20 node problems for k=2

Problem	Optimal value	M1 CPU (Sec)	YM1 CPU (Sec)	YM2 CPU (Sec)	Problem	Optimal value	M1 CPU (Sec)	YM1 CPU (Sec)	YM2 CPU (Sec)
1	1950	379.54	456.47	7.47	11	1697	1990.11	7200	15.35
2	1655	169.15	1369.75	1.23	12	2000	479.47	1589.03	1.73
3	1926	401.82	814.67	1.50	13	1879	1032.91	2576.46	3.37
4	1871	7200	7200	11.30	14	1723	423.04	2131.60	2.00
5	1724	2153.24	7200	9.50	15	1795	172.51	383.68	2.93
6	1887	7200	7200	15.97	16	1989	3879.36	7200	4.15
7	1560	7200	7200	8.46	17	1570	1687.05	7200	2.22
8	1962	7200	7200	6.87	18	1696	7200	7200	14.97
9	1989	7200	7200	20.90	19	1859	162.84	146.14	2.38
10	1948	1489.03	7200	2.66	20	1674	109.04	275.11	1.89
Mean						-	1037.79	1082.55	6.84
Standard deviation						-	1092.27	876.94	5.97

According to CPU times, YM2 is faster than the other formulations for 20 node problems and k=2. Mean and standard deviation of CPU of the YM2 are considerably smaller than the others. Some of these problems cannot be solved in time limit 7200 seconds by the other formulations.

When we look all these values for 10 and 20 node problems and k=2, YM2 model is always superior to the others. For this reason, after this stage the performance of YM2 model is searched by increasing problem size and changing k values. While these analyses are carried out, the benchmark instances for k-TRP used by Luo et al. (2014). In these benchmark instances, there are 6 different data sets named as brd14051, d15112, d18512, fnl4461, nrw1379 and pr1002. Each of these sets includes 10 different data. The node numbers of these problems are 29, 39, 49 as well as one node is depot. Also they use k=6 for 29 node problems, k=8 for 39 node problems and k=10 for 49 node problems.

In our study, YM2 model is solved for all these cases. Also, for doing an experimental analysis about k values, for 29 node problems YM2 model is solved for k=1, k=2 and k=4, for 39 node problems k=4 and k=6 and for 49 node problems k=6 and k=8 additionally. YM2 model can solve all these problems for different k values in time limit 7200 seconds.

In Table 3, the average CPU times for 29 node problems for k=1, k=2, k=4 and k=6 are showed. In Table 4, the average CPU times for 39 node problems for k=4, k=6 and k=8 are listed. In Table 5, the average CPU times for 49 node problems for k=6, k=8 and k=10 are given.

Table 3. Average CPU times for 29 node problems

Problem	Average CPU (Sec)			
	k=1	k=2	k=4	k=6
brd14051	951.02	596.32	26.48	2.22
d15112	770.64	691.86	14.78	2.74
d18512	441.39	272.80	23.00	2.86
fnl4461	902.51	647.07	12.67	2.00
nrw1379	368.30	447.98	20.38	2.56
pr1002	653.71	493.83	24.80	3.05

Table 4. Average CPU times for 39 node problems

Problem	Average CPU (Sec)		
	k=4	k=6	k=8
brd14051	636.98	61.00	13.16
d15112	1855.41	388.28	9.44
d18512	2102.10	86.68	7.77
fnl4461	1271.12	66.74	7.52
nrw1379	829.32	41.81	15.18
pr1002	1520.57	158.88	20.44

Table 5. Average CPU times for 49 node problems

Problem	Average CPU (Sec)		
	k=6	k=8	k=10
brd14051	1480.20	168.71	29.10
d15112	1008.61	129.34	46.75
d18512	2099.08	217.37	31.47
fnl4461	992.95	163.94	23.30
nrw1379	1005.57	171.75	44.59
pr1002	1172.71	244.34	25.27

5. Conclusion

In this paper, new integer linear programming formulation with $O(n^2)$ binary variables and $O(n^2)$ constraints is presented for multiple traveling repairmen problem. We conduct a computational analysis in order to see the performance of the new formulation against the other models by using benchmark instances existing in the literature. We experimentally prove that proposed formulation is superior to other models in terms of CPU times. Besides that, we carried out more analysis to see the performance of proposed model with benchmark instances for k-TRP by increasing problem size. Consequently, we observe that proposed formulation can solve all the k-repairmen benchmark instances optimally using CPLEX 12.6.0.0 as shown tables. In the literature, as far as we know there do not exist many formulations for this problem. For this reason, our adaptation is very important contribution to this area. It is also observed that if the value of k is increased, CPU time is decreased. These models can be adapted to the repairmen problem with time windows as next studies.

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